A panoramic view on buildings

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Introduction

Buildings are mathemtatical objects that have a rich geometric and combinatorial structure and they appear naturally in many mathematical subdisciplines, ranging from number theory over geometric group theory up to analysis. In this seminar we want to develop a panoramic view on buildings, i.e. we want to see how these buldings show up in the different subdisciplines.

Part I: Basics

This part is dedicated to introduce/recall the basic defintions of buildings and it aims to build a common ground (concerning notations etc) for the rest of the seminar.

Talk 1 (Carlo Kaul): Coxeter complexes

Coxeter complexes are natural geometric objects on which Coxeter groups act. Coexter groups are algebraic generalizations of reflection groups. For us (aiming to understand affine bulidngs) in particular the reflection groups of the Euclidean plane are interesting. In this case Coxeter complexes are always tessaltions of the Euclidean space as well as of its sphere at infity (for the corresponding spherical building).

Literatur:

- Brown: Chapter III and Chapter VI.1, VI.2
- Abramenko-Brown: Chapter III

Talk 2 (Daniel Kahl): Buildings as simplicial complexes/chamber complexes

Buildings arise by gluing different copies of coxeter compelxes. This gluing is done in a way such that the resulting object exhibits a large degree of symmetry. It would be nice to get an insight in how this symmetry arises.

Literatur:

- Brown: Chapter 4
- Abramenko-Brown: Chapter 4

Part II: Overview of standard topics of buildings theory

In this part we want to get to know (some!) standard topics in the the theory of buldings.

Talk 3 (Daniel Kahl): BN-Pairs and Flag compelxes for spherical bulidngs

The points in the projective plane over a field $\mathbb{K}$ are realized as one-dimensional subspaces of $\mathbb{K}^3$ and the lines in the projective plane correspond to 2-dimensional subspaces. The relation "A point lies on a line" corresponds to the inclusion of subspaces. Spherical buildings are – from that perspective – generalized projective geometries.

An efficient way to see that such Flag complexes are indeed Buildings is to adopt the view point of strongly transitive automrphism groups and BN-pairs which shall also be introduced in this talk

Literatur:

- Brown: Chapter V, Examples 5 - 7
- Abramenko-Brown: Chapter 6, in particular 6.5 - 6.8

Talk 4 (Radou Toma): Lattice class models for affine buildings

Lattices in $\mathbb{R}^n$ of full rank play a role in constructing the symmetric space of $SL(2,\mathbb{R})$ and $GL(n,\mathbb{R})$. Analoguesly lattices in $\mathbb{Q}_p$ of full rank can be used to construct the affine buildings on which $SL(n,\mathbb{Q}_p)$ acts. They are thus p-adic analoga of symmetric spaces

Literatur:

- Brown: Chapter V, Example 8
- Serre: Chaper II § 1, in particular II.1.1
- Abramenko-Brown: Chapter 6, in particular 6.9

Talk 5 (Kai-Uwe Bux): Reductive groups over local fields: The example SL_2

The ends of the tree associated to $SL(2,\mathbb{Q}_p)$ can be identified with the projective lines over Q_p and the geometry encodes the p-adic valuation. In this talk we like to understand how this picute can be generalized to more geneal fiels with discrete valuations

Talk 6 (Paul Kiefer): Hecke Algebra and the Satake Isomorphism

In this talk we want to introduce different perspectives on Hecke algebras and describe the construction of the Satake Isomorphism. In particular we want to understand how the Hecke algebra acts as a commutative algebra of vertex averaging operators on the buidlings.

Literatur:
tba

Talk 7 (Lasse Wolf): The algebra of invariant differential operators on Riemannian locally symmetric spaces

On Riemannian locally symmetric spaces the counterpart of the Hecke algebra and the Satake Isomorphism are the algebra of invariant differential operators and the Harish-Chandra Isomorphism. In this talk we want to introduce those objects and discuss their spectral theory. In an outlook we will sketch the open spectral theoretic questions studied in the CRC project B2.

Part III: CRC research projects related to buildings

In this final part we want to explain how buldings arise in different research projects of the CRC. The precise schedule will be fixed during the semester. Possible topics are:

Talk 8 (Christian Arends): Poisson Tranformationen on trees of bounded degree (Project B3/B4)

tba

Literatur:
tba

Talk 9 (Margit Rösler): Macdonald spherical functions

The spherical functions of a Bruhat-Tits building can be identified with point evaluations of the so-called Macdonald spherical functions, which form a natural basis of the invariants in the group algebra of the associated coweight lattice. We shall discuss these polynomials in connection with the underlying affine Hecke algebra, as well as some of their properties and representations.

Literatur:

- I.G. Macdonald, Spherical functions on a group of p-adic type, Ramanujan Institute, 1971
- J. Parkinson, Spherical harmonic analysis on affine buildings, Math. Z. 2006
- K. Nelsen, A. Ram, Kostka-Foulkes polynomials and Macdonald spherical functions, London Math. Soc. Lecture Note Ser. 307, 2003

Possible additional topics:

- Quantum classical correspondence on graphs of bounded degree (Project B4)
- Weyl Chamber flows on symmetric spaces and builgings (Project B4)
- Gelfand pairs and hypergroups (Project B3)
- ...what else?

Literatur

[Bro89] Brown K.S.. *Buildings*, Springer, 1989

[AB08] Abramenko P., Brown, K.S.: *Buildings - Theory and Applications* (GTM 248), Springer, 2008.

[Ser02] Serre J.P. *Trees*, Springer, 2002ß