DOKTORANDENSEMINAR WS25/26

The seminar takes place on Mondays from 2 p.m. to 3 p.m. in J2.138.

Topics		
03.11.25	Segal's Plancherel Theorem - part I	L. Langen
10.11.25	Segal's Plancherel Theorem - part II	L. Langen
17.11.25	Segal's Plancherel Theorem - part III	L. Langen
24.11.25	Segal's Plancherel Theorem - part IV	L. Langen
01.12.25	Segal's Plancherel Theorem - part V	L. Langen

Below you find a list of abstracts.

1. Segal's Plancherel Theorem - L. Langen

Plancherel theorems come in many different flavours in harmonic analysis. Typical examples include:

(1) The classical Plancherel theorems establishing isometric isomorphisms $\mathcal{F}\colon L^2(\mathbb{R}) \to L^2(\mathbb{R})$ and $\mathcal{F}\colon L^2(\mathbb{T}) \to \ell^2(\mathbb{Z})$ for the Fourier transform on $\mathbb{R}$ and the Fourier series of periodic signals, i.e. on the torus $\mathbb{T}$.

(2) The Plancherel theorem establishing an isometric isomorphism $\mathcal{F}\colon L^2(G) \to L^2(\widehat{G})$ for the Fourier transform on locally compact abelian groups G with corresponding dual group $\widehat{G} = \{\alpha\colon G \to \mathbb{T} \mid \alpha \text{ continuous group morphism}\}$.

(3) The Peter-Weyl theorem establishing an isometric isomorphism $\mathcal{F}\colon L^2(G) \to \widehat{\bigoplus}_{\pi\in\widehat{G}} \operatorname{End}(V_\pi)$ for the Fourier transform on compact groups G. Here, $\widehat{G}$ denotes the unitary dual of G, i.e. (equivalence classes of) unitary irreducible representations of G.

Note that the case $G = \mathbb{R}$ is covered by (1) and (2), while the case $G = \mathbb{T}$ is covered by (1) and (3). In fact, in these cases the respective Plancherel theorems agree. The common property of all these Fourier transforms $\mathcal{F}$ is that they intertwine the left-regular action $L\colon G \curvearrowright L^2(G)$ with the left-regular action $G \curvearrowright \mathcal{F}(L^2(G))$, i.e. "translation becomes multiplication by a phase factor in the Fourier image". Viewing this intertwining property as the distinctive property of Plancherel theorems, we understand a Plancherel theorem as a disintegration of the regular representation.

We aim to prove Segal's Plancherel theorem ([1]) for locally compact, separable, unimodular, CCR groups G, which disintegrates the regular representation $U\colon G \times G \curvearrowright L^2(G), (x, y).f(z) := f(x^{-1}zy)$ (G acts from the left and the right) into a direct integral of irreducibles:

$$(U,\, L^2(G)) \;\cong\; \left(\int_{\widehat{G}}^{\oplus} \pi \mathbin{\hat{\otimes}} \pi' \; d\zeta(\pi),\; \int_{\widehat{G}}^{\oplus} H_\pi \mathbin{\hat{\otimes}} H'_\pi \; d\zeta(\pi)\right)$$

Date: November 25, 2025.

To reach this goal we will encounter the following topics: the group C^*-algebra, the Jacobson topology on $\widehat{G}$, direct integrals of Hilbert spaces, the spectral theorem for commutative von Neumann algebras, constructing the Plancherel measure ζ.

Segal's Plancherel theorem encompasses the above examples (1)-(3) and many more groups such as real reductive groups.

We will follow the exposition in [2, Ch. 14]. A complete treatise in more generality can be found in [3]. Some conditions (separable, unimodular, CCR) on the group can be weakened, see [3].

Contents of the talks:

Part I: Motivation: Classical Plancherel theorems on $\mathbb{R}, \mathbb{R}/\mathbb{Z}$, LCA groups and compact groups. Basic representation theory of locally compact groups.

Part II: Integrated representations, the group C^*-algebra, Jacobson topology on $\mathrm{Prim}(\mathcal{C}) \cong \widehat{\mathcal{C}}$ for CCR groups.

Part III: Direct integrals of Hilbert spaces, the spectral theorem for commutative von Neumann algebras.

Part IV: Direct integrals of representations, disintegration into irreducible representations and into factor representations, multiplicities.

Part V: The Plancherel theorem.

References

[1] I. E. Segal, "An Extension of Plancherel's Formula to Separable Unimodular Groups." Ann. Math., vol. 52, no. 2, 1950, pp. 272–292.

[2] N. R. Wallach, *Real reductive groups II*, Pure and Applied Mathematics, vol. 132, Academic Press, Boston, MA, 1992.

[3] J. Dixmier, *C^*-algebras*, North-Holland Mathematical Library, vol. 15, North-Holland Publishing Co., Amsterdam-New York-Oxford, 1977, Translated from the French by Francis Jellett.

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