Degenerate Elliptic Boundary Value Problems with Non-smooth Coefficients

Elmar Schrohe

Leibniz Universität Hannover
schrohe@math.uni-hannover.de

On a manifold of bounded geometry with boundary we consider a uniformly strongly elliptic second order operator A that locally is of the form

$$A = -\sum_{j,k} a_{jk}\partial_{x_j}\partial_{x_k} + \sum_j b_j\partial_{x_j} + c$$

together with a degenerate boundary operator T of the form

$$T = \varphi_0\gamma_0 + \varphi_1\gamma_1,$$

where γ_0 and γ_1 denote the evaluation of a function and its exterior normal derivative, respectively, at the boundary, and φ_0, φ_1 are smooth functions on the boundary with $\varphi_0 \geq 0$, $\varphi_1 \geq 0$ and $\varphi_0 + \varphi_1 \geq c_0 > 0$. Unless either $\varphi_0 \equiv 0$ or $\varphi_1 \equiv 0$ this problem is not elliptic in the sense of Lopatinskij and Shapiro.

We show that the realization A_T of A in $L^p(\Omega)$ has a bounded H^∞-calculus whenever the a_{jk} are Hölder continuous and b_j as well as c are L^∞. For the proof we first treat the operator with smooth coefficients on $\mathbb{R}^n_+$. Here we rely on an extension of Boutet de Monvel's calculus to operator-valued symbols of Hörmander type $(1,\delta)$. We then use H^∞-perturbation techniques in order to treat the non-smooth case.

As an application we study the porous medium equation.

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