ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO THE HEAT EQUATION ON NONCOMPACT SYMMETRIC SPACES

E. PAPAGEORGIOU

ABSTRACT. Consider the heat equation on $\mathbb{R}^n$,

$$\partial_t u(t,x) = \Delta_x u(t,x), \quad u(0,x) = f(x),$$

with initial data $f \in L^1(\mathbb{R}^n)$. Denote by $M = \int_{\mathbb{R}^n} f(x)\,dx$ the mass and by $h_t(x) = (4\pi t)^{-\frac{n}{2}} e^{-\frac{\|x\|^2}{4t}}$ the heat kernel. Then the following asymptotics are known to hold in $L^p(\mathbb{R}^n)$, for all $1 \leq p \leq \infty$:

$$\lim_{t\to+\infty} t^{\frac{n}{2p'}} \|u(t,\cdot) - M\,h_t\|_{L^p(\mathbb{R}^n)} = 0$$

Analogous heat asymptotics may or may not hold on Riemannian manifolds. Our aim is to discuss noncompact symmetric spaces, generalizing earlier results of J.L. Vázquez on real hyperbolic spaces. Interestingly, both with the Laplace-Beltrami operator and with the distinguished Laplacian, some non-euclidean phenomena occur.

Joint work with J.-Ph. Anker (Université d'Orléans) and H.-W. Zhang (Ghent University).

Current address: University of Paderborn