

## SEMINAR SOPHUS LIE

46th Session, January 10–11, 2014

Institut für Mathematik, Universität Paderborn  
Warburger Str. 100, 33098 Paderborn

## PROGRAM

### Friday, January 10

9:00-9:05	Welcome and opening (Lecture room D2)
9:05-9:55	<i>Michel Duflo:</i> On Frobenius parabolic Lie subalgebras of $sl(n)$
10:00-10:50	<i>Gert Heckman:</i> Remarkable ball quotients
10:50-11:20	Coffee break
11:20-12:10	<i>Angela Pasquale:</i> Resonances on Riemannian symmetric spaces: the case of $SL(3, \mathbb{R})/SO(3)$
12:10-13:30	Lunch (on Campus)
13:30-14:20	<i>Job Kuit:</i> Cusp forms for reductive symmetric spaces
14:25-15:15	<i>Piotr Graczyk:</i> Convolution of orbital measures on symmetric spaces of type $B_p, C_p$ and $D_p$
15:15-15:45	Coffee break
16:00	Excursion and Dinner

### Saturday, January 11

9:00-9:50	<i>Jasper Stokman:</i> Affine Hecke algebras and the XXZ spin chain
9:55-10:45	<i>Jan Möllers:</i> Knapp-Stein type intertwiners for symmetric pairs
10:45-11:10	Coffee break
11:10-12:00	<i>Yuri Neretin:</i> Infinite symmetric groups and simplicial bordisms
12:05-12:25	<i>Timothee Marquis:</i> One-parameter subgroups of Kac-Moody groups
12:30-12:50	<i>Karl Leicht:</i> Kähler structures on $T^*G$ and invariant theory
12:50-14:30	Lunch and coffee
14:30-15:20	<i>Ilka Agricola:</i> On the classification of naturally reductive homogeneous spaces in small dimensions
15:25-16:15	<i>Michael Pevzner:</i> Rankin-Cohen Operators for symmetric pairs

**Locations:** Talks take place in lecture hall **D2**, Building D  
Coffee breaks in room **D2.343** (2nd floor of building D, blue area)

# ABSTRACTS

## On the classification of naturally reductive homogeneous spaces in small dimensions

Ilka Agricola (Universität Marburg)

A homogeneous space is naturally reductive if it carries a metric connection with skew torsion that has parallel torsion and parallel curvature - they are hence a natural generalisation of symmetric spaces. Based on recent progress on the holonomy theory of metric connections with skew torsion, I shall present our work on the classification of naturally reductive homogeneous spaces of dimension less or equal to 6 by endowing them with an induced geometric structure. We explain how this approach differs from the classical techniques of Kowalski and Vanhecke, who made this classification up to dimension 5 in the mid 80ies, and we discuss a few particularly interesting examples.

This is joint work with Ana Cristina Ferreira (Minho/Portugal) and Thomas Friedrich (Humboldt-Universität zu Berlin).

## On Frobenius parabolic Lie subalgebras of $\mathfrak{sl}(n)$

Michel Duflo (Université Paris Diderot-Paris 7)

I will present some results about the number of parabolic and biparabolic Lie subalgebras of  $sl(n)$ .

## Convolution of orbital measures on symmetric spaces of type $B_p$ , $C_p$ and $D_p$

Piotr Graczyk (Université d'Angers)

This is a common work with P. Sawyer. We study the absolute continuity of the convolution  $\delta_{e^X}^\sharp \star \delta_{e^Y}^\sharp$  of two orbital measures on noncompact Grassmannians  $\mathbf{SO}_0(p, q)/\mathbf{SO}(p) \times \mathbf{SO}(q)$  as well as on the the symmetric spaces  $\mathbf{SO}_0(p, p)/\mathbf{SO}(p) \times \mathbf{SO}(p)$ ,  $\mathbf{SU}(p, p)/\mathbf{S}(\mathbf{U}(p) \times \mathbf{U}(p))$  and  $\mathbf{Sp}(p, p)/\mathbf{Sp}(p) \times \mathbf{Sp}(p)$ . We prove sharp conditions on  $X, Y \in \mathfrak{a}$  for the existence of the density of the convolution measure. This measure intervenes in the product formula for the spherical functions.

## Remarkable ball quotients

**Gert Heckman** (Radboud University Nijmegen)

We shall discuss complex ball quotients  $B/\Gamma$ , for which  $\Gamma$  is generated by complex reflections. Subsequently we review ball quotients related to point configurations on a line (Appell-Lauricella hypergeometric functions), next ball quotients related to point configurations in a plane (exceptional root system hypergeometric functions), and finally two ball quotients found by Daniel Allcock for which a modular interpretation is still missing. We shall provide additional evidence for the monstrous proposal of Allcock.

## Cusp forms for reductive symmetric spaces

**Job Kuit** (University of Copenhagen)

This is joint work with Erik van den Ban and Henrik Schlichtkrull. For a real reductive Lie group  $G$ , there exists a notion of cusp form, which was introduced by Harish-Chandra. He showed that the space of cusp forms coincides with the discrete part of the spectral decomposition of the space of square integrable functions on  $G$ . The class of real reductive symmetric spaces contains the real reductive Lie groups. It would be interesting to have a notion of cusp form for this class of spaces, but the generalization of Harish-Chandra's definition turns out to be somewhat problematic due to the fact that certain integrals are divergent. Some years ago, Flensted-Jensen suggested a definition, that led to the ongoing work with Erik van den Ban and Henrik Schlichtkrull. In this talk I will discuss the problems with convergence of the integrals and propose a modification of Flensted-Jensen's idea that gives a solution for symmetric spaces of split rank 1.

## Kähler structures on $T^*G$ and invariant theory

**Karl Leicht** (University of Lille 1)

For a compact connected Lie group  $G$ , we construct  $G$ -biinvariant Kaehler structures on the total space  $T^*G$  of the cotangent bundle of  $G$  using invariant theory. In particular, we exhibit a Kaehler structure that is not diffeomorphic to the standard one obtained by identifying  $T^*G$  with  $G^{\mathbb{C}}$  in the standard fashion. We discuss some related questions.

## One-parameter subgroups of Kac-Moody groups

**Timothée Marquis** (Universität Erlangen-Nürnberg)

Kac-Moody groups may be viewed as infinite-dimensional analogues of semi-simple Lie groups (or rather, semi-simple algebraic groups). In a broad sense, a Kac-Moody group is a group attached to a Kac-Moody Lie algebra, where a Kac-Moody algebra is some infinite-dimensional Lie algebra which possesses, as for semi-simple (finite-dimensional) Lie algebras, a Cartan decomposition with respect to some set of roots.

Although Kac-Moody groups may be turned into Hausdorff topological groups, one does not know (beyond the affine case) how to equip them with a Lie group structure. In fact, the Lie correspondence between a Kac-Moody group and its Lie algebra is very poorly understood. On the other hand, Kac-Moody groups possess a very rich algebraic structure and act naturally in a nice way on some geometric objects called buildings: this provides a very powerful tool to study them.

In this talk, I will illustrate how these geometric tools allow to better understand the Lie correspondence between a real or complex Kac-Moody group  $G$  and its Lie algebra by describing the set of continuous one-parameter subgroups of  $G$ . At the end of my talk, I will also briefly explain my main current research project, which in some sense tries to fill the gap between Kac-Moody groups and infinite-dimensional Lie groups.

## Knapp-Stein type intertwiners for symmetric pairs

**Jan Möllers** (Aarhus University)

For a symmetric pair  $(G, H)$  we construct a family of intertwining operators between spherical principal series representations of  $G$  and  $H$  that are induced from parabolic subgroups satisfying certain compatibility conditions. The operators are given explicitly in terms of their integral kernels and we prove convergence of the integrals for an open set of parameters and meromorphic continuation. For the rank one cases  $(G, H) = (U(1, n; \mathbb{F}), U(1, m; \mathbb{F}) \times U(n-m; \mathbb{F}))$ ,  $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ , we further show that these operators generically span the space of intertwiners.

## Infinite symmetric groups and simplicial bordisms

**Yuri Neretin** (University of Vienna and Moscow State University)

We produce representations of categories of bordisms from unitary representations of infinite symmetric groups.

## **Resonances on Riemannian symmetric spaces: the case of $SL(3, \mathbb{R})/SO(3)$**

**Angela Pasquale** (Université de Lorraine, Metz)

Let  $\Delta$  be the Laplace-Beltrami operator on a Riemannian symmetric space of the non-compact type  $G/K$ , and let  $\sigma(\Delta)$  denote its spectrum. The resolvent  $R(z) = (\Delta - z)^{-1}$  is a holomorphic function on  $\mathbb{C} \setminus \sigma(\Delta)$ , with values in the space of bounded operators on  $L^2(G/K)$ . We study the meromorphic continuation of  $R$  as distribution valued map on a Riemann surface above  $\mathbb{C} \setminus \sigma(\Delta)$ . If such a meromorphic continuation is possible, then the poles of the meromorphically extended resolvent are called the resonances. The main questions are the existence, location and interpretation of resonances. This talk, based on a joint work with Joachim Hilgert and Tomasz Przebinda, presents the case of  $SL(3, \mathbb{R})/SO(3)$ .

## **Rankin-Cohen Operators for symmetric pairs**

**Michael Pevzner** (Université de Reims)

The celebrated Rankin-Cohen brackets can be interpreted as intertwining operators between different unitary representations of the Lie group  $SL(2, \mathbb{R})$ . We present their analogue arising in the setting of breaking symmetries for six different complex geometries and explain in a systematic way why such differential operators are related to Jacobi polynomials in one variable.

## **Affine Hecke algebras and the XXZ spin chain**

**Jasper Stokman** (University of Amsterdam and Radboud University Nijmegen)

The Heisenberg XXZ spin chain is a well known integrable one-dimensional quantum lattice model. Solutions of the associated quantum Knizhnik-Zamolodchikov (KZ) equations give rise to correlation functions. I will construct a basis of solutions and will describe the monodromy of the quantum KZ equations in terms of explicit elliptic solutions of dynamical Yang-Baxter and reflection equations.

At the basis of these results is the observation that the spin chains are governed by Baxterizations of principal series modules of affine Hecke algebras. This allows one to use the theory on double affine Hecke algebras and on basic hypergeometric functions associated to root systems in this context.

## **PARTICIPANTS**

Alexander Alldridge	Mathematisches Institut, Universität zu Köln, Weyertal 86-90, 50931 Köln, Germany; alldridg@math.uni-koeln.de
Ilka Agricola	Philipps-Universität Marburg, FB 12 / Mathematik und Informatik, Hans-Meerwein-Str./Campus Lahnberge 35032 Marburg, Germany; agricola@mathematik.uni-marburg.de
Nils Byrial Andersen	Department of Mathematics, Aarhus University, Ny Munkegade 118, DK-8000 Aarhus, Denmark; byrial@imf.au.dk
Simon Chiossi	Phillips-Universität Marburg, FB 12 / Mathematik und Informatik, Hans-Meerwein-Str./Campus Lahnberge 35032 Marburg, Germany; simon.chiossi@polito.it
Rafael Dahmen	Fachbereich Mathematik, Technische Universität Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany; dahmen@mathematik.tu-darmstadt.de
Michel Duflo	Université Paris-Diderot (Paris 7), UFR de Mathématiques - Bâtiment Sophie Germain, 5 rue Thomas Mann, 75205 Paris CEDEX 13, France; duflo@math.jussieu.fr
Jan Emonds	Insitut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; emonds@math.upb.de
Christian Fleischhack	Insitut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; christian.fleischhack@math.uni-paderborn.de
Walter Freyn	TU Darmstadt, Schlossgartenstr. 7, 64289 Darmstadt, Germany; freyn@mathematik.tu-darmstadt.de
Thomas Friedrich	Institut für Mathematik, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany; friedric@mathematik.hu-berlin.de
Helge Glöckner	Insitut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; glockner@math.upb.de
Piotr Graczyk	LAREMA Université d'Angers, 2 bd. Lavoisier, F-49045 Angers, France; graczyk@univ-angers.fr
Gert Heckman	Radboud University Nijmegen, Department of Mathematics, P.O.Box 9010, 6500 GL Nijmegen, The Netherlands; G.Heckman@math.ru.nl
Joachim Hilgert	Insitut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; hilgert@math.uni-paderborn.de
Troels Johansen	Mathematisches Seminar, Chr.-Albrechts-Universität Kiel, Ludewig-Meyn-Strasse 4, 24098 Kiel, Germany; johansen@math.uni-kiel.de
Oguzhan Kaya	University Lille 1, Cité Scientifique Université de Sciences et Tech. de Lille 1, 59655 Villeneuve d'Ascq Cedex, France; oguzabel@gmail.com
Bernhard Kroetz	Insitut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; bkroetz@math.upb.de
Job Kuit	University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark; j.j.kuit@gmail.com

Deniz Kus	University of Cologne, Mathematical Institute, Weyertal 86-90, Cologne; dkus@math.uni-koeln.de
Karl Leicht	Université Lille 1, Lille 1, France; karl.leicht@math.univ-lille1.fr
Gang Liu	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; gliu@math.uni-paderborn.de
Timothée Marquis	Department Mathematik, FAU Erlangen-Nürnberg, Cauerstr. 11, 91058 Erlangen, Germany; marquis@math.fau.de
Jan Möllers	Department of Mathematics, Aarhus University, Denmark, Ny Munkegade 118, DK-8000 Aarhus, Denmark; moellers@imf.au.dk
Karl-Hermann Neeb	Department Mathematik, FAU Erlangen-Nürnberg, Cauerstr. 11, 91058 Erlangen, Germany; neeb@math.fau.de
Yuri Neretin	Department of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Wien, Austria; neretin(at)mccme.ru
Dominik Ostermayr	Mathematisches Institut, Universität zu Köln, Weyertal 86-90, 50931 Köln, Germany; dominik.ostermayr@gmail.com
Aprameyan Parthasarathy	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; apram@math.upb.de
Angela Pasquale	Université de Lorraine, Institut Elie Cartan de Lorraine, Bâtiment A, Ile dy Saulcy, 57045 Metz cedex 1, France; angela.pasquale@univ-lorraine.fr
Michael Pevzner	University of Reims, Campus Moulin de la Housse, BP 1039, Reims, 51687 France; pevzner@univ-reims.fr
Maarten van Pruijssen	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; vanpruijssen@math.uni-paderborn.de
Tomasz Przebinda	University of Oklahoma, Department of Mathematics, 601 Elm Street, Physical Sciences Building, Norman, OK 73019, USA; tprzebinda@gmail.com
Margit Rösler	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; roesler@math.upb.de
Martin Schlichenmaier	University of Luxembourg, Mathematics Research Unit, FSTC, Campus Kirchberg, 6, rue Coudenhove-Kalergi, L-1359 Luxembourg-Kirchberg; martin.schlichenmaier@uni-lu
Alexander Schmeding	Institutt for matematiske fag, NTNU Gløshaugen Trondheim, NTNU Gløshaugen Sentralbygg 2, Alfred Getz v 1, 7491 Trondheim; alexander.schmeding@math.ntnu.no
Andreas Schmied	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; aschmied@math.uni-paderborn.de
Benjamin Schwarz	Institut für Mathematik, Universität Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; bschwarz@math.upb.de
Henrik Seppänen	Mathematisches Institut, Georg-August-Universität Göttingen, Bunsenstraße 3-5, 37073 Göttingen, Germany; hseppaen@uni-math.gwdg.de

Thomas Skill	Hochschule Bochum, Lennershofstr. 140, 44801 Bochum, Germany; ThomasSkill@hs-bochum.de
Jasper Stokman	KdV Institute for Mathematics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands; j.v.stokman@uva.nl
Aleksander Strasburger	Department of Applied Mathematics, Warsaw University of Life Sciences–SGGW, Nowoursynowska 159, 02-776 Warsaw, Poland; aleksander.strasburger@sggw.pl
Valdemar Tsanov	Ruhr-Universität Bochum, Fakultät für Mathematik, 44780 Bochum, Germany; valdemar.tsanov@rub.de
Michael Voit	Fakultät für Mathematik, TU Dortmund, Vogelpothsweg 87, 44227 Dortmund, Germany; michael.voit@math.tu-dortmund.de
Christoph Wockel	Fachbereich Mathematik, Universität Hamburg, Bundesstr. 55, 20146 Hamburg, Germany; christoph@wockel.eu
Oksana Yakimova	Mathematisches Institut, Friedrich-Schiller-Universität Jena, 07737 Jena, Germany; oksana.yakimova@uni-jena.de