Disturbance decoupling for descriptor systems by behavioral feedback

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We study disturbance decoupling for linear descriptor systems. Compared to previous approaches, where state feedback is used, we use the concept of behavioral feedback which allows to study a larger class of systems. We derive geometric characterizations for solvability of the disturbance decoupling problem following the classical approach. Exploiting the freedom in the choice of the behavioral feedback we show that whenever disturbance decoupling can be achieved by behavioral feedback we may additionally achieve autonomous zero dynamics.

1 Disturbance decoupled systems

We study disturbance decoupling for linear descriptor systems governed by differential-algebraic equations (DAEs),

$$E\dot{x}(t) = Ax(t) + Bu(t)\,, \quad y(t) = Cx(t)\,, \tag{1.1}$$

where $E, A \in \mathbb{R}^{l\times n}$, $B \in \mathbb{R}^{l\times m}$, $C \in \mathbb{R}^{p\times n}$. The set of systems (1.1) is denoted by $\Sigma_{l,n,m,p}$ and we write $[E, A, B, C] \in \Sigma_{l,n,m,p}$. Note that we do not assume regularity of the pencil $sE - A$. The tuple $(x, u, y) : \mathbb{R} \to \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p$ is said to be a *solution* of (1.1), if it belongs to the *behavior* of (1.1):

$$\mathfrak{B}_{[E,A,B,C]} := \left\{ \begin{array}{l} (x,u,y) \in \\ \mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^p) \end{array} \middle| (x,u,y) \text{ satisfies (1.1)} \right\}.$$

Based on the above behavior, DAE control systems have been studied in detail e.g. in [1]. We assume that the states, inputs and outputs of the systems in $\Sigma_{l,n,m,p}$ are fixed a priori by the designer. This is different from other approaches based on the behavioral setting, see [2–4].

For given $Q \in \mathbb{R}^{l\times q}$ we consider the disturbed system

$$E\dot{x}(t) = Ax(t) + Bu(t) + Qd(t)\,, \quad y(t) = Cx(t)\,, \tag{1.2}$$

where $d \in \mathcal{C}^\infty(\mathbb{R} \to \mathbb{R}^q)$ represents a smooth disturbance, which may be due to noise, modeling or measuring errors, or higher terms in linearization.

Definition 1.1 For a system $[E, A, B, C] \in \Sigma_{l,n,m,p}$, we call the set-valued map

$$\Phi_{[E,A,B,C]} : \mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^m) \to \mathcal{P}\big(\mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^p)\big),$$
$$u \mapsto \left\{ y \in \mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^p) \,\middle|\, \begin{array}{l} \exists\, x \in \mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^n) : \\ (x,u,y) \in \mathfrak{B}_{[E,A,B,C]} \end{array} \right\},$$

the *input-output map* of $[E, A, B, C]$. Here $\mathcal{P}(\mathcal{M})$ denotes the power set of a set $\mathcal{M}$.

Definition 1.2 Let $[E, A, 0, C] \in \Sigma_{l,n,0,p}$ and $Q \in \mathbb{R}^{l\times q}$. Then we call $[E, A, Q, C]$ *disturbance decoupled*, if

$$\forall\, w_1, w_2 \in \mathcal{C}^\infty(\mathbb{R}\to\mathbb{R}^q) : \quad \Phi_{[E,A,Q,C]}(w_1) = \Phi_{[E,A,Q,C]}(w_2).$$

Roughly speaking, $[E, A, Q, C]$ is disturbance decoupled, if any two disturbances cannot be distinguished using knowledge of the output.

The crucial geometric tools for the characterization of disturbance decoupling are the *generalized Wong sequences*

$$\begin{aligned} \mathcal{V}^0_{[E,A,B,C]} &= \ker C, \\ \mathcal{V}^{i+1}_{[E,A,B,C]} &= A^{-1}(E\mathcal{V}^i_{[E,A,B,C]} + \operatorname{im} B) \cap \ker C, \\ \mathcal{W}^0_{[E,A,B,C]} &= \{0\}, \\ \mathcal{W}^{i+1}_{[E,A,B,C]} &= E^{-1}(A\mathcal{W}^i_{[E,A,B,C]} + \operatorname{im} B) \cap \ker C. \end{aligned}$$

The sequence $(\mathcal{V}^i_{[E,A,B,C]})_{i\in\mathbb{N}_0}$ is non-increasing and $(\mathcal{W}^i_{[E,A,B,C]})_{i\in\mathbb{N}_0}$ is non-decreasing and both sequences terminate after finitely many steps, thus we may set

$$\mathcal{V}^*_{[E,A,B,C]} = \bigcap_{i\in\mathbb{N}_0} \mathcal{V}^i_{[E,A,B,C]}, \; \mathcal{W}^*_{[E,A,B,C]} = \bigcup_{i\in\mathbb{N}_0} \mathcal{W}^i_{[E,A,B,C]}.$$

In [6–8] the Wong sequences for matrix pencils ($B = 0$ and $C = 0$) are investigated, the name chosen this way since Wong [9] was the first who used both sequences for the analysis of matrix pencils. In [3, 10, 11] the case $C = 0$ is considered and the sequences are called *augmented Wong sequences*; similarly, for the case $B = 0$ considered in [12] the sequences are called *restricted Wong sequences*.

Theorem 1.3 *Let* $[E, A, 0, C] \in \Sigma_{l,n,0,p}$, $Q \in \mathbb{R}^{l\times q}$. *Then*

$$\begin{array}{ll} & [E, A, Q, C] \text{ is disturbance decoupled} \\ \iff & \operatorname{im} Q \subseteq E\mathcal{V}^*_{[E,A,0,C]} + A\mathcal{W}^*_{[E,A,0,C]}. \end{array}$$

For the proofs and more details we refer to [5].

2 Disturbance decoupling

In this section we consider the disturbance decoupling problem (DDP) by the application of feedback. The classical result, see [13, Thm. 4.2], states that there exists $F \in \mathbb{R}^{m\times n}$ such that $[I, A + BF, Q, C]$ is disturbance decoupled if, and only if, $\operatorname{im} Q \subseteq \mathcal{V}^*_{[I,A,B,C]}$. This has been generalized to DAEs in [14] (see also [15, 16]) using proportional state feedback of the form $u = Fx$ as well. However, using the behavioral approach we find that the input variables are not the

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free variables of the system, i.e., some of them may already be constrained, and on the other hand free state variables may be present. Therefore, the use of proportional state feedback is limited for DAE systems, and actually a feedback in terms of the free variables is needed. A setup where this is allowed is provided by the use of *behavioral feedback* of the form $K_1x+K_2u=0$, where $K=[K_1,K_2]\in\mathbb{R}^{k\times n}\times\mathbb{R}^{k\times m}$. The interconnection of system (1.2) with the behavioral feedback is depicted in Figure 1.

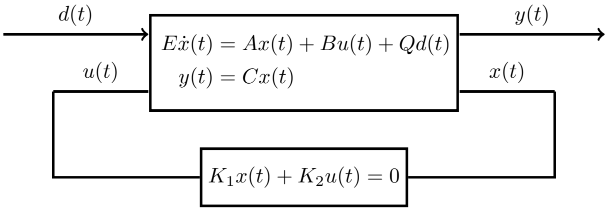

Fig. 1: Interconnection of system and behavioral feedback

The closed-loop system of (1.2) with the behavioral feedback $K_1x+K_2u=0$ is given by

$$[E^K,A^K,Q^K,C^K]=\left[\begin{bmatrix}E&0\\0&0\end{bmatrix},\begin{bmatrix}A&B\\K_1&K_2\end{bmatrix},\begin{bmatrix}Q\\0\end{bmatrix},[C,0]\right]$$

with state $\binom{x}{u}$, input d and output y. If $[K_1,K_2]=[F,-I_m]$, then $[E^K,A^K,Q^K,C^K]$ is equivalent to $[E,A+BF,Q,C]$ and we are in the case of proportional state feedback.

Definition 2.1 Let $[E,A,B,C]\in\Sigma_{l,n,m,p}$. A control matrix $K=[K_1,K_2]\in\mathbb{R}^{k\times n}\times\mathbb{R}^{k\times m}$ is called *compatible* for $[E,A,B,C]$, if

$$\forall\,(x,u,y)\in\mathfrak{B}_{[E,A,B,C]}\;\exists\,(\tilde{x},\tilde{u})\in\mathfrak{B}_{[E^K,A^K,0_{l\times 0},0_{0\times n}]}:\quad Ex(0)=E\tilde{x}(0).$$

Theorem 2.2 *Let* $[E,A,B,C]\in\Sigma_{l,n,m,p}$ *and* $Q\in\mathbb{R}^{l\times q}$. *Then there exists a control* $K=[K_1,K_2]\in\mathbb{R}^{k\times n}\times\mathbb{R}^{k\times m}$ *compatible for* $[E,A,B,C]$ *such that* $[E^K,A^K,Q^K,C^K]$ *is disturbance decoupled if, and only if,*

$$\boxed{\operatorname{im}Q\subseteq E\mathcal{V}^*_{[E,A,B,C]}+A\mathcal{W}^*_{[E,A,B,C]}+\operatorname{im}B.}$$

In fact, in Theorem 2.2 the freedom in choosing the control $[K_1,K_2]$ is not exploited. Therefore, we consider additional properties and introduce the zero dynamics defined by

$$\mathcal{ZD}_{[E,A,B,C]}\;:=\;\left\{\,(x,u,y)\in\mathfrak{B}_{[E,A,B,C]}\;\middle|\;y=0\,\right\}.$$

For linear DAE systems the zero dynamics have been well investigated, see [17–20]. The zero dynamics of (1.1) are called *autonomous*, if

$$\forall\,w\in\mathcal{ZD}_{[E,A,B,C]}\;\forall\,I\subseteq\mathbb{R}\text{ open intvl.}:\;w|_I=0\;\Rightarrow\;w=0.$$

Theorem 2.3 *Let* $[E,A,B,C]\in\Sigma_{l,n,m,p}$ *and* $Q\in\mathbb{R}^{l\times q}$. *Then there exists a control* $K=[K_1,K_2]\in\mathbb{R}^{k\times n}\times\mathbb{R}^{k\times m}$ *compatible for* $[E,A,B,C]$ *such that* $[E^K,A^K,Q^K,C^K]$ *is disturbance decoupled and* $\mathcal{ZD}_{[E^K,A^K,0,C^K]}$ *are autonomous if, and only if,*

$$\operatorname{im}Q\subseteq E\mathcal{V}^*_{[E,A,B,C]}+A\mathcal{W}^*_{[E,A,B,C]}+\operatorname{im}B.$$

The behavioral feedback approach to disturbance decoupling opens the door for the study of various related problems and extensions such as disturbance decoupled state estimation and disturbance decoupling by dynamic feedback controllers. In the absence of disturbances these problems have already been treated using the framework of behavioral feedback, see [10, 21].

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