Funnel control for nonlinear functional DAE systems

Thomas Berger, Achim Ilchmann, Timo Reis

Fachbereich Mathematik, Universität Hamburg

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ODE systems

$$\dot{x}(t) = f_1(x(t), y(t)), \qquad x(0) = x^0, \quad (1)$$
$$\dot{y}(t) = f_2(y(t)) + f_3(x(t)) + \Gamma(y(t))u(t), \qquad y(0) = y^0. \quad (2)$$

$x : \mathbb{R} \to \mathbb{R}^n$, $u, y : \mathbb{R} \to \mathbb{R}^m$; f_1, f_2, f_3 differentiable;
$\Gamma : \mathbb{R}^m \to \mathbb{R}^{m \times m}$ differentiable

(1) internal dynamics
(2) input-output behavior

functional: $x = T(y)$, $T : \mathcal{C} \to \mathcal{C}^1$

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Relative degree:

$$\dot{z}(t) = f(z(t)) + g(z(t))u(t)$$
$$y(t) = h(z(t))$$

has **strict relative degree** $r \in \mathbb{N} \;:\Longleftrightarrow$ [Isidori, 1995]

- $\forall\, \xi \in \mathbb{R}^n \;\forall\, k = 0, \ldots, r-2: \; L_g L_f^k h(\xi) = 0,$
  $[L_f h = (\partial h / \partial z)\, f]$
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(1), (2) has str. rel. deg. 1 $\Longleftrightarrow \det \Gamma(y) \neq 0 \;\forall\, y \in \mathbb{R}^m$

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Zero dynamics:

$$\begin{aligned} \dot{z}(t) &= f(z(t)) + g(z(t))u(t) \\ y(t) &= h(z(t)) \end{aligned}$$

$$\mathcal{ZD} := \{\ (z,u)\ \mid \dot{z}(t) = f(z(t)) + g(z(t))u(t),\ \ 0 = h(z(t))\ \};$$

$$\mathcal{ZD}\ \textbf{stable}\ :\Longleftrightarrow\ \forall\,(z,u) \in \mathcal{ZD}:\ \lim_{t\to\infty}(z(t), u(t)) = 0$$

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$\mathcal{ZD}_{(1),(2)}$ stable, if (1) **input-to-state stable**: [Sontag, 1989]

$$\exists\,\alpha \in \mathcal{KL}, \beta \in \mathcal{K}\ \forall\,(x^0, y) \in \mathbb{R}^n \times \mathcal{C}^0(\mathbb{R}_{\geq 0}; \mathbb{R}^m)$$
$$\forall\,t \geq 0:\ \|x(t; x^0, y)\| \leq \alpha(\|x^0\|, t) + \sup_{s\in[0,t]} \beta(\|y(s)\|),$$

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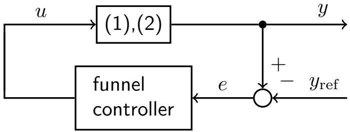

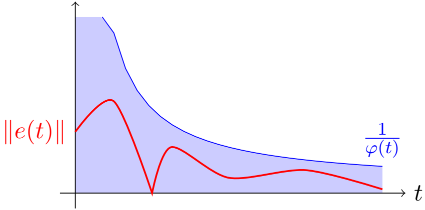

[Ilchmann & Ryan, 2009]: Funnel control is feasible if

- strict relative degree 1 and "high-gain Matrix" $\Gamma(y)$ pos. def. for all $y \in \mathbb{R}^m$
- (1) is ISS (zero dynamics are stable)

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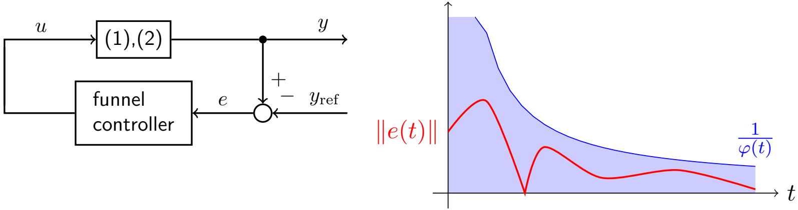

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$$\{(1),(2) \text{ with str. rel. deg. } 1\} \overset{\tilde\Gamma = \Gamma^{-1}}{\subseteq} \{(3),(4)\}$$

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$$\dot{x}(t) = x(t) + y_1(t)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \end{pmatrix} = \begin{pmatrix} 0 \\ y_2(t) \\ y_3(t) \end{pmatrix} + \begin{pmatrix} x(t) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{pmatrix}$$

$$y_1 = u_1 - \dot{u}_1 \quad \Longrightarrow \quad \int y_1 = \int u_1 - u_1$$

$$G(s) = \begin{bmatrix} s-1 & 0 & 0 \\ 0 & -1 & \\ 0 & 0 & \frac{1}{s-1} \end{bmatrix}, \qquad \lim_{s\to\infty} \begin{bmatrix} s^{-1} & 0 & 0 \\ 0 & s^{0} & 0 \\ 0 & 0 & s^{1} \end{bmatrix} G(s) \in \mathbb{R}^{3\times 3} \text{ inv.}$$

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Theorem (funnel control)

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- (3) is ISS
- $\tilde{\Gamma}(y) = RG(y)R^\top$, where $G(y) > 0 \;\forall\, y \in \mathbb{R}^m$
- $\operatorname{im} K = \ker R^\top$ and $K^\top f_2' K$ bounded
- $\hat{k} > \|(K^\top K)^{-1}\| \cdot \sup_{y\in\mathbb{R}^m} \|K^\top f_2'(y) K\|$
- y^0 such that $K^\top\big(f_2(y^0) + f_3(x^0) - \hat{k}(y^0 - y_{\mathrm{ref}}(0))\big) = 0$

Then the *funnel controller*

$$u(t) = -k(t)\,e(t), \qquad \text{where} \qquad e(t) = y(t) - y_{\mathrm{ref}}(t)$$
$$k(t) = \hat{k}/\big(1 - \varphi(t)^2\|e(t)\|^2\big),$$

achieves: $(x, y, k) \in L^\infty, \;\wedge\; \exists\, \varepsilon > 0\; \forall\, t > 0 : \; \|e(t)\| \le \varphi(t)^{-1} - \varepsilon$

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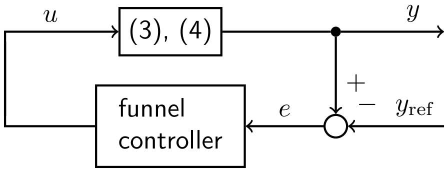

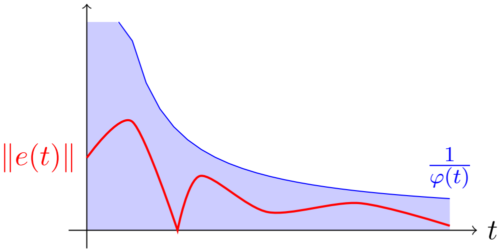

$\hat{k}$-condition

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$$y(t) = [K, R]\begin{bmatrix}(K^\top K)^{-1}K^\top \\ (R^\top R)^{-1}R^\top\end{bmatrix} y(t) = [K, R]\begin{pmatrix} y_1(t) \\ y_2(t)\end{pmatrix}$$

$$R^\top(4)\colon (R^\top R)G(y(t))R^\top[K,R]\begin{pmatrix}\dot{y}_1(t)\\ \dot{y}_2(t)\end{pmatrix} = (R^\top R)G(y(t))(R^\top R)\dot{y}_2(t) = \ldots$$

$$K^\top(4)\colon 0 = K^\top\big(f_2'(y(t))\dot{y}(t) + f_3'(x(t))f_1(x(t),y(t)) - \dot{k}(t)e(t) - k(t)\dot{e}(t)\big)$$

$$\Rightarrow\ 0 = \underbrace{\Big(K^\top f_2'(y(t))K - k(t)\big(K^\top K + H(t, y_1(t), y_2(t))\big)\Big)}_{=:T}\dot{y}_1(t) + \psi(t, y_1(t), y_2(t), \dot{y}_2(t))$$

$$H \geq 0,\ k(t) \geq \hat{k} > \|(K^\top K)^{-1}\| \cdot \sup_{y\in\mathbb{R}^m} \|K^\top f_2'(y)K\| \quad \Rightarrow \quad T \text{ inv.}$$

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Comparison of ODEs and DAEs

$$\begin{aligned} \dot x(t) &= f_1(x(t), y(t)), & x(0) &= x^0, \quad (1)\\ \dot y(t) &= f_2(y(t)) + f_3(x(t)) + \Gamma(y(t))u(t), & y(0) &= y^0. \quad (2) \end{aligned}$$

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ODEs	DAEs
(1) is ISS	(3) is ISS
$\Gamma > 0$ (rel. deg. 1)	$\tilde\Gamma \geq 0$ (mixed rel. deg.)
$\hat k = 1$	$\hat k > \\|(K^\top K)^{-1}\\| \cdot \sup_{y\in\mathbb{R}^m} \\|K^\top f_2'(y)K\\|$
$y^0 \in \mathbb{R}^m$	$K^\top\big(f_2(y^0) + f_3(x^0) - \hat k(y^0 - y_{\mathrm{ref}}(0))\big) = 0$