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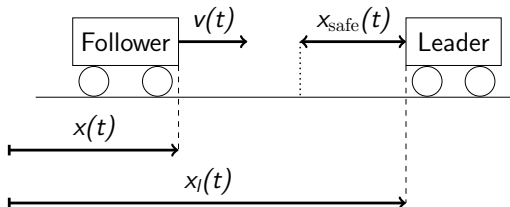
FAKULTÄT
FÜR MATHEMATIK, INFORMATIK
UND NATURWISSENSCHAFTEN

FACHBEREICH MATHEMATIK

THOMAS BERGER, ANNA-LENA RAUERT

Funnel cruise control

Vehicle following framework



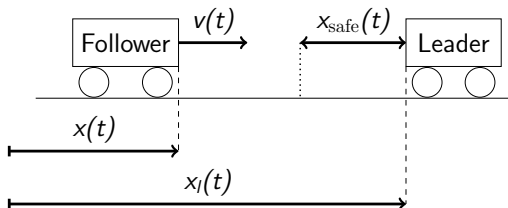
$$\dot{x}(t) = v(t), \quad m\dot{v}(t) = u(t) - F_g(t) - F_a(t, v(t)) - F_r(v(t))$$

forces due to gravity: $F_g(t) = mg \sin \theta(t)$

aerodynamic drag: $F_a(t, v) = \frac{1}{2} \rho(t) C_d A v^2$

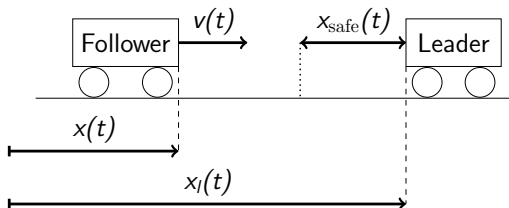
rolling friction: $F_r(v) = mg C_r \text{sgn}(v)$

Control objective



given: favourite speed $v_{\text{ref}}(t)$, safety distance $x_{\text{safe}}(t) = \lambda_1 v(t) + \lambda_2$

Control objective



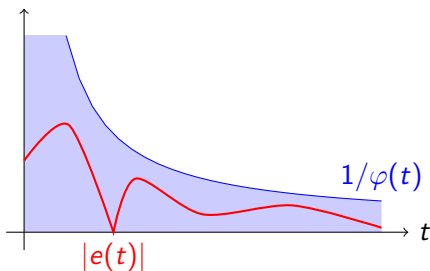
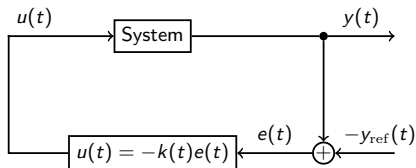
given: favourite speed $v_{\text{ref}}(t)$, safety distance $x_{\text{safe}}(t) = \lambda_1 v(t) + \lambda_2$

Aim: $u(t) = F(t, v(t), x_l(t) - x(t))$ s.t.

(O1) $x_l(t) - x(t) \geq x_{\text{safe}}(t),$

(O2) $|v(t) - v_{\text{ref}}(t)|$ is as small as possible such that (O1) is not violated.

Funnel Control



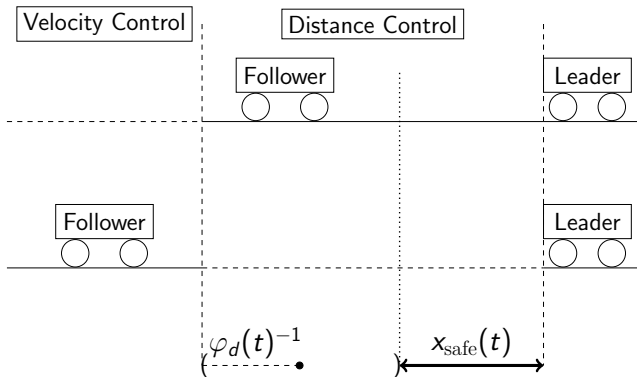
[ILCHMANN, RYAN, SANGWIN '02]:

Works, if

- relative degree = 1
- stable internal dynamics

$$k(t) = \frac{1}{1 - \varphi(t)^2 e(t)^2}$$

Funnel cruise control



Velocity funnel control

tracking error: $e_v(t) = v(t) - v_{\text{ref}}(t) \in \mathcal{F}_v(t) := (-\varphi_v(t)^{-1}, \varphi_v(t)^{-1})$

→ treat $v(t)$ as output of the system

$$\dot{v}(t) = \frac{1}{m}u(t) + f_v(t, v(t))$$

controller:

$$u_v(t) = -k_v(t)e_v(t),$$

$$k_v(t) = \frac{1}{1 - \varphi_v(t)^2 e_v(t)^2}$$

Distance funnel control

tracking error:
$$e_d(t) = x(t) - x_l(t) + x_{\text{safe}}(t) + \varphi_d(t)^{-1}$$
$$\in \mathcal{F}_d(t) := (-\varphi_d(t)^{-1}, \varphi_d(t)^{-1})$$

Distance funnel control

tracking error:
$$e_d(t) = x(t) - x_I(t) + x_{\text{safe}}(t) + \varphi_d(t)^{-1}$$

$$\in \mathcal{F}_d(t) := (-\varphi_d(t)^{-1}, \varphi_d(t)^{-1})$$

output:
$$y(t) = \lambda_1 v(t) + x(t),$$

reference signal:
$$y_{\text{ref}}(t) = x_I(t) - \lambda_2 - \varphi_d(t)^{-1}$$

Distance funnel control

tracking error:
$$e_d(t) = x(t) - x_I(t) + x_{\text{safe}}(t) + \varphi_d(t)^{-1}$$

$$\in \mathcal{F}_d(t) := (-\varphi_d(t)^{-1}, \varphi_d(t)^{-1})$$

output:
$$y(t) = \lambda_1 v(t) + x(t),$$

reference signal:
$$y_{\text{ref}}(t) = x_I(t) - \lambda_2 - \varphi_d(t)^{-1}$$

$$\dot{x}(t) = v(t) = -\frac{1}{\lambda_1}x(t) + \frac{1}{\lambda_1}y(t),$$

$$\dot{y}(t) = \frac{\lambda_1}{m}u(t) + f_d(t, x(t), y(t))$$

controller:
$$u_d(t) = -k_d(t)e_d(t),$$

$$k_d(t) = \frac{1}{1 - \varphi_d(t)^2 e_d(t)^2}$$

Final control design

Intuitive approach:

- if $e_d(t) = -\varphi_d(t)^{-1}$,
then switch from u_v to
 u_d and vice versa
- while u_d is active it
should be possible that
 $e_v(t) < -\varphi_v(t)^{-1}$, but
not $e_v(t) \geq \varphi_v(t)^{-1}$

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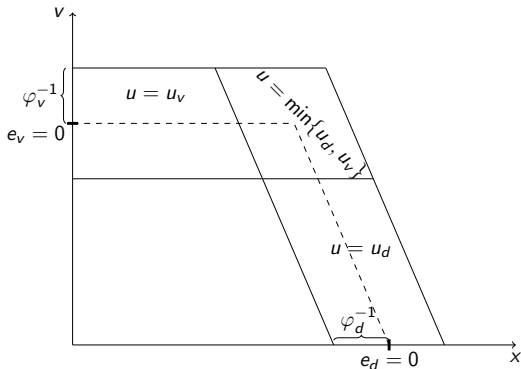
Problem:

- when $e_d(t) \rightarrow -\varphi_d(t)^{-1}$ we have $k_d(t) \nearrow \infty$
- when $e_v(t) \rightarrow -\varphi_v(t)^{-1}$ we have $k_v(t) \nearrow \infty$

Final control design

Intuitive approach:

- if $e_d(t) = -\varphi_d(t)^{-1}$, then switch from u_v to u_d and vice versa
- while u_d is active it should be possible that $e_v(t) < -\varphi_v(t)^{-1}$, but not $e_v(t) \geq \varphi_v(t)^{-1}$

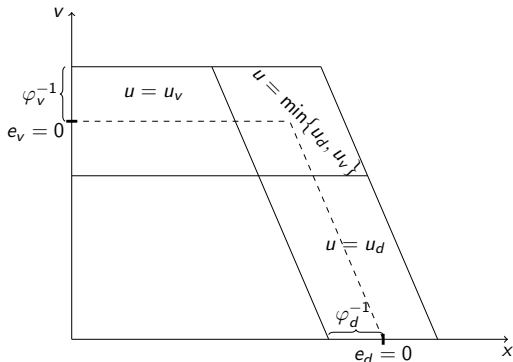


Problem:

- when $e_d(t) \rightarrow -\varphi_d(t)^{-1}$ we have $k_d(t) \nearrow \infty$
- when $e_v(t) \rightarrow -\varphi_v(t)^{-1}$ we have $k_v(t) \nearrow \infty$

Final control design

$$u(t) = \begin{cases} u_v(t), & e_d(t) \leq -\varphi_d(t)^{-1} \wedge e_v(t) \in \mathcal{F}_v(t), \\ u_d(t), & e_v(t) \leq -\varphi_v(t)^{-1} \wedge e_d(t) \in \mathcal{F}_d(t), \\ \min\{u_v(t), u_d(t)\}, & e_v(t) \in \mathcal{F}_v(t) \wedge e_d(t) \in \mathcal{F}_d(t), \end{cases}$$



Final control design

$$u(t) = \begin{cases} u_v(t), & e_d(t) \leq -\varphi_d(t)^{-1} \wedge e_v(t) \in \mathcal{F}_v(t), \\ u_d(t), & e_v(t) \leq -\varphi_v(t)^{-1} \wedge e_d(t) \in \mathcal{F}_d(t), \\ \min\{u_v(t), u_d(t)\}, & e_v(t) \in \mathcal{F}_v(t) \wedge e_d(t) \in \mathcal{F}_d(t), \end{cases}$$

Theorem [B., RAUERT '18]

$v_{\text{ref}}, x_l \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R})$, $x_{\text{safe}} = \lambda_1 v(t) + \lambda_2$
 $\implies x, v, u \in L^\infty$ and

$$\varphi_v(t)^{-1} - e_v(t) \geq \varepsilon, \quad \varphi_d(t)^{-1} - e_d(t) \geq \varepsilon,$$

$$\max\{0, \varphi_v(t)^{-1} + e_v(t)\} + \max\{0, \varphi_d(t)^{-1} + e_d(t)\} \geq \varepsilon$$

Simulation

m	$\theta(t)$	$\rho(t)$	C_d	C_r	A
1300 kg	0 rad	1.3 kg/m ³	0.32	0.01	2.4 m ²

x^0	v^0	λ_1	λ_2	v_{ref}
0 m	15 m s ⁻¹	0.5 s	2 m	36 m s ⁻¹

$$\varphi_v(t) = (22.5e^{-0.2t} + 0.2)^{-1}, \quad \varphi_d(t) = 0.25.$$

