Single element changes in electrical networks

Thomas Berger,

George Halikias and **Nicos Karcanias** (City University London)

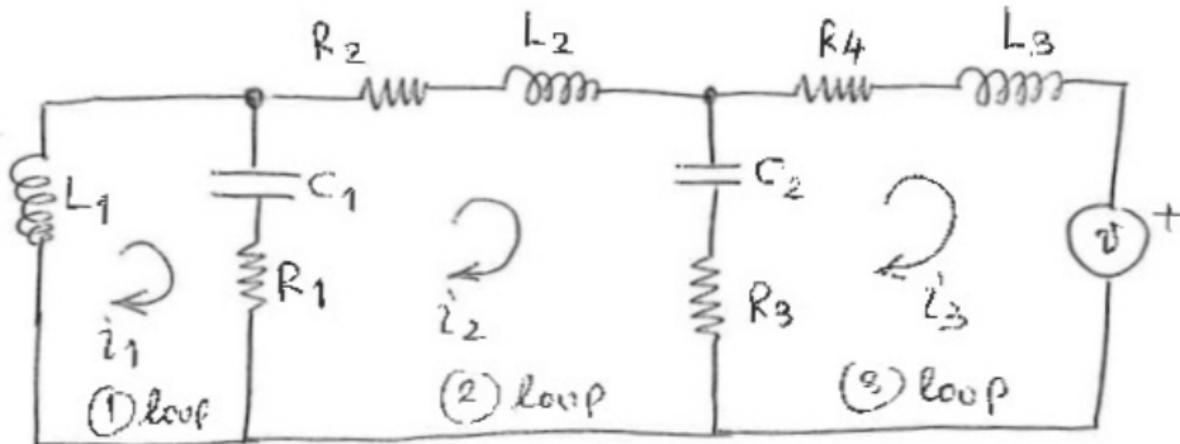Institute of Mathematics, Ilmenau University of Technology

London, February 22, 2012

Single element changes in electrical networks

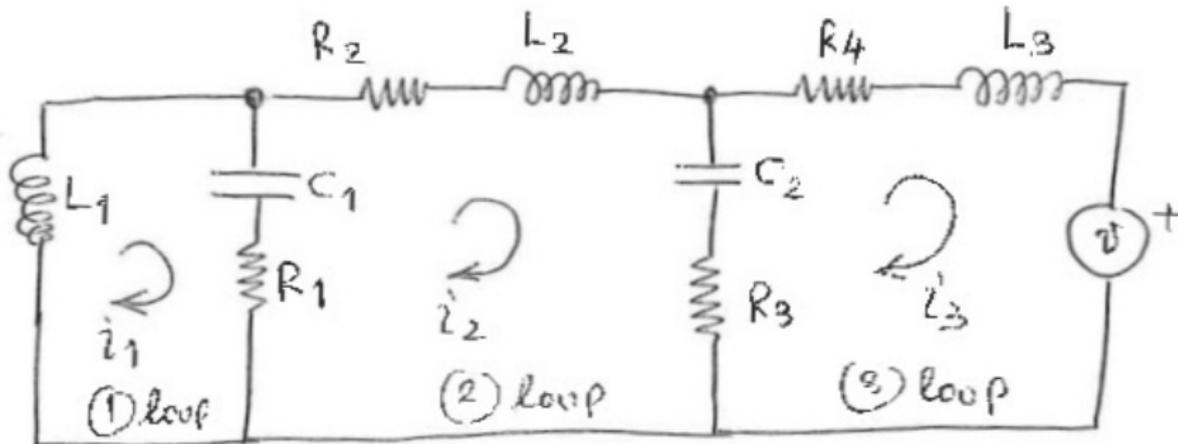Thomas Berger, George Halikias and Nicos Karcanias (City University London)
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Example: electrical RLC network

Single element changes in electrical networks   Modeling of networks

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Example: electrical RLC network

$$Z(s)i(s) = v_s(s),$$

$$Z(s) = s^{-1} \begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} & 0 \\ -\frac{1}{C_1} & \frac{1}{C_1}+\frac{1}{C_2} & -\frac{1}{C_2} \\ 0 & -\frac{1}{C_2} & \frac{1}{C_2} \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix} + s \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix}$$

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RLC networks: impedance/admittance modeling

$$Z(s)i(s) = v_s(s), \quad Y(s)v(s) = i_s(s)$$

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RLC networks: impedance/admittance modeling

$$Z(s)i(s) = v_s(s), \quad Y(s)v(s) = i_s(s)$$

$$Z(s), Y(s): \ W(s) = sL + s^{-1}C + R$$

impedance modeling (loop analysis):

L: mass, spring, inductor
C: inertor, capacitor
R: damper, resistor

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RLC networks: impedance/admittance modeling

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impedance modeling (loop analysis):

L: mass, spring, inductor
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- element present in i-th loop (node) $\Rightarrow$ its value is added to (i,i) position of the respective matrix

- element common to i-th and j-th loop (node) $\Rightarrow$ its value is added to (i,i) and (j,j) positions, substracted from (i,j) and (j,i) positions

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Problem: effect of single element changes on the corresponding natural frequencies of the network

Single element changes in electrical networks   Problem statement

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Problem: effect of single element changes on the corresponding natural frequencies of the network

special cases: RC and RL networks $\rightarrow W(s) = sL + s^{-1}C + R$ becomes symmetric matrix pencil:

$$W(s) = \left\{ \begin{array}{l} \text{RL: } sL + R \\ \text{RC: } \hat{s}C + R, \ \hat{s} = s^{-1} \end{array} \right\} = sF + G, \quad F = F^\top, G = G^\top$$

Single element changes in electrical networks

Problem statement

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$$W(s) = s \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix}$$

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$$W(s) = s \begin{bmatrix} L_1+L_4 & -L_4 & 0 \\ -L_4 & L_2+L_4 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4+R_5 \end{bmatrix}$$

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$$
\begin{aligned}
W(s) &= s \begin{bmatrix} L_1{+}L_4 & -L_4 & 0 \\ -L_4 & L_2{+}L_4 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1{+}R_2{+}R_3 & -R_3 \\ 0 & -R_3 & R_3{+}R_4{+}R_5 \end{bmatrix} \\
&= s \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1{+}R_2{+}R_3 & -R_3 \\ 0 & -R_3 & R_3{+}R_4 \end{bmatrix} \\
&\quad + sL_4(e_1 - e_2)(e_1 - e_2)^\top + R_5 e_3 e_3^\top
\end{aligned}
$$

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&= s(F + L_4 b_1 b_1^\top) + (G + R_5 b_2 b_2^\top)
\end{aligned}
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&= s(F + L_4 b_1 b_1^\top) + (G + R_5 b_2 b_2^\top)
\end{aligned}
$$

here: $\boxed{\det\left(s(F + x b b^\top) + G\right)}$ $\quad b = e_i$ or $b = e_i - e_j, \ i \neq j$

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$$W(s) = s \begin{bmatrix} L_1+L_4 & -L_4 & 0 \\ -L_4 & L_2+L_4 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix}$$

Single element changes in electrical networks   Root locus problem

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$$\det W(s) = \underbrace{\det W_{L_4=0}(s)}_{=p(s)} + sL_4 z(s)$$

$$z(s) = \big((sL_2 + R_2 + R_3)(sL_3 + R_3 + R_4)$$
$$+ sL_1(sL_3 + R_3 + R_4) - R_3^2\big)$$

Single element changes in electrical networks   **Root locus problem**

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$$W(s) = s \begin{bmatrix} L_1+L_4 & -L_4 & 0 \\ -L_4 & L_2+L_4 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix}$$

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$$\det W(s) = 0 \quad \Longleftrightarrow \quad 1 + L_4 \frac{sz(s)}{p(s)} = 0$$

Single element changes in electrical networks   Root locus problem

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Reformulation as root locus problem [Binet-Cauchy-Theorem]

$$\det\left(s(F + xbb^\top) + G\right) = \det\left([sF + G, I_k]\begin{bmatrix} I_k \\ sxbb^\top \end{bmatrix}\right)$$

$$= g(s; F, G)^\top p(sx; b), \qquad x \in \mathbb{R}$$

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\end{aligned}
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$$
\det\left(s(F + xbb^\top) + G\right) = \det(sF + G) + sx\, z(s; b)
$$

root locus problem:
$$
\boxed{1 + x\,\frac{s\, z(s; b)}{\det(sF + G)} = 0}
$$

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Root locus problem

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Theorem (Properties of the root locus)

$$\boxed{1 + x\,\frac{s\,z(s;b)}{\det(sF+G)} = 0}\quad x \in \mathbb{R}, \quad \textbf{Assumption}: G \text{ is invertible}$$

- all branches of root locus are restricted to the real axis; for $x > 0$ all branches are restricted to the negative real axis

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Root locus problem

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- $\exists\, p < 0 \; \exists\, \mu \in \mathbb{N}, \mu \geq 2 : \; \det(sF + G) = \eta(s)(s - p)^{\mu} \implies z(s;b) = \zeta(s)(s - p)^{\mu-1}$

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- $\mathrm{rk}(F + bb^{\top}) = \mathrm{rk}\, F + 1 \implies \deg \det(sF + G) = \deg z(s;b)$

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- $\operatorname{rk}(F + bb^{\top}) = \operatorname{rk} F + 1 \implies \deg\det(sF+G) = \deg z(s;b)$

- $\operatorname{rk}(F + bb^{\top}) = \operatorname{rk} F \implies \deg\det(sF+G) = \deg z(s;b) + 1$

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$$F + xbb^\top = \begin{bmatrix} C_1 + x & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

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Root locus problem

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$$F + xbb^\top = \begin{bmatrix} C_1 + x & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

$$\det(sF + G) + x\,sz(s;b) = 1/R_2(s(C_1 + x) + 1/R_1)$$

$$p(x) = -1/(R_1(C_1 + x)) :$$

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$$p(x) = -1/(R_1(C_1 + x)) :$$

$-C_1 < x < 0$ $\quad$ $-1/(R_1C_1)$ $\quad$ $x > 0$ $\quad$ 0 $\quad$ $x < -C_1$

$$\lim_{x \searrow -C_1} p(x) = -\infty \qquad \lim_{x \to \infty} p(x) = 0 \qquad \lim_{x \nearrow -C_1} p(x) = \infty$$

$$\text{"}p(-C_1) = \infty\text{"} \qquad\qquad\qquad\qquad\qquad \text{"}p(-C_1) = \infty\text{"}$$

Single element changes in electrical networks $\qquad\qquad$ Root locus problem

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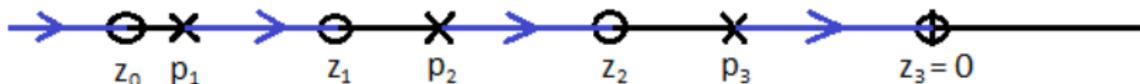ILMENAU

Theorem (Behaviour of the root locus)

$x > 0$:

- p pole of multiplicity $\mu \implies \mu$ or $\mu - 1$ of these poles do not change and at most one pole moves to the right
- $\exists$ pole $p(x)$ s.t. $\lim_{x \to \infty} p(x) = 0$
- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\, F$:
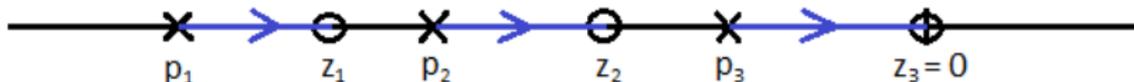
- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\, F + 1$, then an infinite pole becomes finite and moves to the right as x increases

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Root locus problem

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$$F + xbb^\top = \begin{bmatrix} C_1 & 0 \\ 0 & x \end{bmatrix}, \quad G = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

Single element changes in electrical networks   Root locus problem

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$$F + xbb^\top = \begin{bmatrix} C_1 & 0 \\ 0 & x \end{bmatrix}, \quad G = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

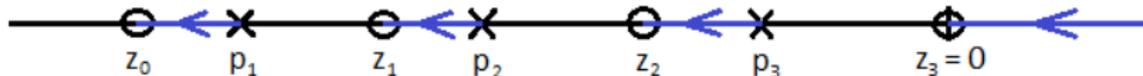$$\det(sF + G) + x\, sz(s; b) = (sC_1 + 1/R_1)(sx + 1/R_2)$$

$$\det(sF + G) = 1/R_2(sC_1 + 1/R_1) \implies p_1 = -1/(C_1R_1)$$
$$sz(s) = sx(sC_1 + 1/R_1) \implies z_1 = -1/(C_1R_1), \; z_2 = 0$$

$p_1 = z_1$ $p(x) = -1/(xR_2)$ $z_2 = 0$

Single element changes in electrical networks   Root locus problem

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$x < 0$:

- p pole of multiplicity $\mu \implies \mu$ or $\mu - 1$ of these poles do not change and at most one pole moves to the left

- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\, F + 1$, then an infinite pole becomes finite and moves to the left on the positive real axis as x decreases, reaching 0 for $x \to -\infty$

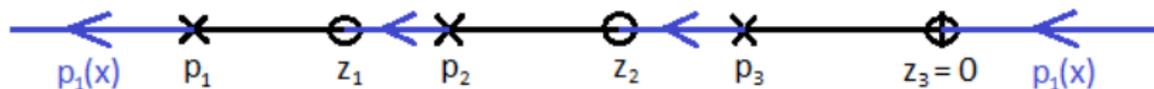Single element changes in electrical networks

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- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\, F$, then the smallest pole $p_1(x)$ moves to the left towards $-\infty$ and

$$\exists\, \kappa > 0: \ \forall -\kappa < x < 0: \ p_1(x) < p_1(0) \ \wedge \ \lim_{x \searrow -\kappa} p_1(x) = -\infty,$$
$$\text{``}p(-\kappa) = \infty\text{''},$$
$$\forall\, x < -\kappa: \ p_1(x) > 0 \ \wedge \ \lim_{x \to -\infty} p_1(x) = 0$$

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Summary

RLC networks: special cases of RC and RL $\rightarrow$ natural frequencies

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Summary

RLC networks: special cases of RC and RL $\rightarrow$ natural frequencies

single element changes $\rightarrow$ root locus problem $1 + x \frac{s\,z(s;b)}{\det(sF+G)} = 0$

Single element changes in electrical networks

Root locus problem

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Summary

RLC networks: special cases of RC and RL $\rightarrow$ natural frequencies

single element changes $\rightarrow$ root locus problem $1 + x \frac{s\,z(s;b)}{\det(sF+G)} = 0$

$x > 0$:
- all poles move to the right
- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\,F + 1$, then an infinite pole becomes finite

$x < 0$:
- all poles move to the left
- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\,F + 1$, then an infinite pole becomes finite
- if $\mathrm{rk}(F + bb^\top) = \mathrm{rk}\,F$, then a finite pole becomes infinite for a single value of x

Single element changes in electrical networks

Root locus problem

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