13th International Conference on p-adic Functional Analysis

August 12–16, 2014

University of Paderborn

13th International Conference on *p*-adic Functional Analysis

Overview

	Tuesday August 12	Wednesday August 13	Thursday August 14	Friday August 15	Saturday August 16
9:00 a.m.	Perez Garcia	Pulita	Lindahl	Zuniga	Boutabaa
	Olivos	FV Kuhlmann	Kakol/Kubzdela	Rodriguez	Lee
	coffee	*coffee*	*coffee*	*coffee*	*coffee*
	Keller	Nagel	Shamseddine	Diarra	Maïga
	Nova/Aguayo	Giordano	Lapidus	Khrennikov	Glöckner
	lunch	*lunch*	*lunch*	*lunch*	*lunch*
2:00 p.m.	Escassut	Aguayo	Ziemkowska	Ojeda	
	coffee	*coffee*	*coffee*	*coffee*	
3:15 p.m.	K Kuhlmann	Diagana	Ivankov	Wittig	
	Berz	Sergeyev	Belhadef	Menken	
				Kuzmenko	
7:15 p.m.	*conference dinner*				

All talks take place in the lecture hall D2 (Hörsaal D2) on the ground floor of building D

Tea and coffee in the coffee breaks is available in room D2.343 on the second floor of building D (i.e., two floors above ground level)

Lunch Tuesday–Friday in the university cafeteria *Mensa*, building *ME*, first floor (i.e., one floor above ground level)

Lunch Saturday: Restaurant *Shangrila*
Warburger Straße 138
33100 Paderborn
Phone: 05251-6938698

Conference dinner: Restaurant *Ratskeller*
Rathausplatz 1
33098 Paderborn
Phone: 05251-201133

Tuesday, August 12, 2014

09:00–10:00	Cristina Perez-Garcia	
	Wim Schikhof: Our colleague and friend	
10:10–10:55	Elena Olivos	
	A note on Banach spaces over a rank 1 *discretely valued field*	
10:55–11:25	*Tea and coffee*	
11:25–12:10	Hans Keller	
	Large affine spaces of invertible operators	
12:20–13:05	Miguel Nova / Jose Aguayo-Garrido	
	Spectral theory and a spectral measure for a finite-dimensional Banach space	
13:05–14:00	Lunch	
14:00–14:45	Alain Escassut	
	Growth order, type and cotype of p-adic entire functions	
14:45–15:15	Tea and coffee	
15:15-16:00	Katarzyna Kuhlmann	
	Ball spaces – a framework for fixed point theorems inspired by ultrametric spaces I	
16:10–16:55	Martin Berz	
	Spectral theory and linear algebra over Levi-Civita vector- and Hilbert spaces	
19:15	Conference dinner, restaurant *Ratskeller*	

Wednesday, August 13, 2014

09:00–09:45 Andrea Pulita

p-adic differential equations on curves

09:55–10:40 Franz-Viktor Kuhlmann

Ball spaces – a framework for fixed point theorems inspired by ultrametric spaces II

10:40–11:10 *Tea and coffee*

11:10–11:55 Enno Nagel

p-adic Taylor polynomials

12:05–12:50 Paolo Giordano
Theory of infinitely near points in smooth manifolds: the Fermat functor

12:50–14:00 Lunch

14:00–14:45 Jose Aguayo

Spectral theory and a spectral measure for an infinite-dimensional Banach space

14:45–15:15 Tea and coffee

15:15-16:00 Toka Diagana

Spectral analysis for some finite rank perturbations of diagonal linear operators in non-archimedean Hilbert space E_ω

16:10–16:55 Yaroslav D. Sergeyev

The infinity computer and numerical computations with infinities and infinitesimals

Thursday, August 14, 2014

09:00–09:45	Karl-Olof Lindahl
	Localization of periodic points and minimally ramified power series
09:55–10:40	Albert Kubzdela / Jerzy Kakol
	Non-archimedean Grothendieck and Krein's theorems
10:40–11:10	*Tea and coffee*
11:10–11:55	Khodr Shamseddine
	New results on the Lebesgue-like measure and integration theory on the Levi-Civita field and applications
12:05–12:50	Michel Lapidus
	p-adic fractal strings: fractal tube formulas, zeta functions and complex dimensions
12:50–14:00	Lunch
14:00–14:45	Agnieszka Ziemkowska
	On complemented subspaces of non-archimedean generalized power series spaces
14:45–15:15	Tea and coffee
15:15-16:00	Nikolay Ivankov
	On bivariant K-theory for ultrametric Banach algebras
16:10–16:55	Rafik Belhadef
	Transcendence of Thue Morse p-adic continued fractions

Friday, August 15, 2014

09:00–09:45	Wilson Zuniga
	Nonlocal operators, parabolic-type equations, and ultrametric random walks
09:55–10:40	John Jaime Rodriguez
	Fundamental solutions of pseudo-differential equations associated with quadratic forms
10:40–11:10	*Tea and coffee*
11:10–11:55	Bertin Diarra
	Ultrametric continuous linear representations of the compact groups $\mathrm{SL}(2, \mathbf{Z}_p)$ $\mathrm{GL}(2, \mathbf{Z}_p)$
12:05–12:50	Andrei Khrennikov
	p-adic Numbers from superstrings and molecular motors to cognition and psychology
12:50–14:00	Lunch
14:00–14:45	Jacqueline Ojeda
	Hayman's conjecture in a non-archimedean context
14:45–15:15	Tea and coffee
15:15-15:45	Alexander Wittig
	Superlinear convergent iterative methods on non-archimedean fields and their applications
15:55–16:20	Hamza Menken
	Some properties of Liouville numbers in the non-archimedean case
16:30–17:15	Yuri Kuzmenko
	A nonarchimedean counterpart of Johnson's theorem for discrete groups

Saturday, August 16, 2014

09:00–09:45	Abdelbaki Boutabaa
	Ultrametric q-difference equations and q-Wronskian
09:55–10:40	Junghun Lee
	J-stability of immediately expanding polynomial maps in non-Archimedean dynamics
10:40–11:10	*Tea and coffee*
11:10–11:55	Hamadoun Maïga
	New identities and congruences for Euler numbers
12:05–12:50	Helge Glöckner
	Exponential laws for spaces of ultrametric differentiable functions and applications
13:00	Lunch, restaurant *Shangrila*

Abstracts

Transcendence of Thue Morse p-adic continued fractions

Rafik Belhadef

It has been proved that real numbers defined as a limit of continued fractions with a Thue-Morse sequence as the sequence of positive integer partial quotients are transcendental. The same holds for more general sequences of partial quotients. Moreover transcendence results have been proved for p-adic numbers defined by Hensel development. Here we tackle the transcendence question in case of p-adic numbers defined as a limit of continued fractions by proving a similar result as the first one.

Answering a question of Michel Mendes France, Martine Queffelec first proved in [MQ] the transcendence of the real numbers that are defined as a limit of continued fractions where the sequence of positive integer partial quotients is Thue-Morse sequence. Some years later, Boris Adamczewski and Yann Bugeaud renewed the ideas and proved more general results concerning real numbers defined by their b-adic expansion or their continued fractions development and to p-adic numbers defined by their Hensel expansion (see [ABI, ABII]). Building on this work, we establish similar results in the case of development in p-adic continued fractions.

The subspace theorem is a multidimensional generalization of the theorem of Roth due to Wolfgang Schmidt (1970). It considers only Archimedean absolute value and was generalized by Schlickewei to several absolute values. These are powerful tool in order to show results on the transcendence of real and p-adic numbers (for details see the overview of Michel Waldschmidt).

A theorem for the p-adic case. Using the p-adic version of the subspace theorem, we give sufficient conditions for a number defined through a continued fraction to be quadratic or transcendental.

Theorem. *Let p be a prime odd positive integer. Let $\frac{\alpha_1}{\alpha_2}$ and $\frac{\beta_1}{\beta_2}$ be two rational numbers in $\mathbb{Z}\left[\frac{1}{p}\right] \cap (0;p)$ such that $v_p(\alpha_1) = v_p(\beta_1) = 0$ and $v_p(\alpha_2) \geq v_p(\beta_2) \geq 1$. Let θ be defined in $\mathbb{Q}_p$ as the limit of $[0; a_1, a_2, ...]$ where $a_i \in \{\alpha, \beta\}$. Suppose that the sequence of partial quotients $(a_i)_{i\geq 1}$ is a Thue-Morse word. Let us denote $\Xi = \mathrm{Max}\{\alpha; \beta\}$.*

If $p^{\frac{5v_p(\beta_2)-v_p(\alpha_2)}{6}} > \mathrm{Max}\{\alpha_2; \beta_2\} \times \frac{\Xi+\sqrt{\Xi^2+4}}{2}$, then θ is either transcendental or quadratic.

(Joint work with Alex Esbelin and Tahar Zerzaihi)

References

[ABI] B. Adamczewski, Y. Bugeaud. *On the complexity of algebraic numbers, I. Expansions in integer bases*, Annals of Math. 165 (2007), 547–565.

[ABII] B. Adamczewski, Y. Bugeaud. *On the complexity of algebraic numbers, II. Continued fractions*, Acta Math. 195 (2005) 1–20.

[MQ] M. Queffélec. *Transcendance des fractions continues de Thue-Morse.* J. Number Theory 73 (1998) 201-211.

Spectral theory and linear algebra over Levi-Civita vector- and Hilbert spaces

Martin Berz

We begin our discussion with advanced concepts of linear algebra on finite dimensional vector spaces over the Levi-Civita field. Introducing a natural valuation on the space of linear operators, we derive a fixed point theorem based on this valuation that allows the computation of the solution to many problems up to any prescribed infinitely small error in a finite number of iterations. We utilize this fixed point theorem to develop frequently used tools of linear algebra over the Levi-Civita field. In particular, we introduce the concept of the condition number of a regular matrix, and parallel to the real case, we develop theorems relating to the inverse matrix and its determination by iteration in the case of finitely bounded condition number, which different from the real case allows to rigorously formulate concepts of ill-conditionedness. We then derive theorems pertaining to properties of eigenvalues and eigenvectors of matrices that allow the determination of the spectrum of a matrix using iterative methods, parallel to the use of such methods in the real case.

The last result is the stepping stone towards a spectral theory of Hermitian linear operators. We define the concept of Hermiticity, compactness and regularity for such operators. Then we show the existence of a single eigenvalue via iteration as a generalization of the earlier treated finite dimensional case. Proceeding to the complement space, the procedure is repeated; and proceeding by induction, we obtain the desired result in a similar fashion as in the conventional case.

(Joint work with Alexander Wittig)

Ultrametric q-difference equations and q-Wronskian

A. Boutabaa

Let $\mathbb{K}$ be an ultrametric complete and algebraically closed field and let q be an element of $\mathbb{K}$ which is not a root of unity and is such that $|q| = 1$. In this article, we establish some inequalities linking the growth of generalized q-wronskians of a finite family of elements of $\mathbb{K}[[x]]$ to the growth of the ordinary q-wronskian of this family of power series.

We then apply these results to study some q-difference equations with coefficients in $\mathbb{K}[x]$. Specifically, we show that the solutions of such equations are rational functions.

(Joint work with R. Bouabdelli and B. Belaïdi)

Spectral analysis for some finite rank perturbations of diagonal linear operators in non-archimedean Hilbert space $\mathbb{E}_\omega$

Toka Diagana

In this talk we study the spectral analysis for the class of linear operators in the form $A = D + F$ where D is a (bounded) diagonal linear operator and F is a finite rank operator in the non-archimedean Hilbert space $\mathbb{E}_\omega$. A few examples will also be discussed.

Ultrametric continuous linear representations of the compact groups $\mathbf{SL(2, \mathbb{Z}_p)}$ and $\mathbf{GL(2, \mathbb{Z}_p)}$

Bertin Diarra

Let $SL(2, \mathbb{Z}_p)$ and $GL(2, \mathbb{Z}_p)$ be the special and general linear 2×2 matrix groups, with entries in the ring of p-adic integers $\mathbb{Z}_p$. These groups are finitely generated profinite groups. With the aid of the decomposition in products of elements of these groups (Litoff decomposition for the special linear group), we shall show that any ultrametric continuous linear representation for the special linear group is determined uniquely by two continuous linear operators (resp. four continuous linear operators for the general linear group) submitted to some relations and some topological properties.

(Joint work with Tongobè Mounkoro)

Growth order, type and cotype of p-adic entire functions

Alain Escassut

Let $\mathbb{K}$ be an algebraically closed p-adic complete field of characteristic zero. We define the order of growth $\rho(f)$ and the type of growth $\sigma(f)$ of an entire function $f(x) = \sum_{n=0}^{\infty} a_n x^n$ on $\mathbb{K}$ f as done on $\mathbb{C}$ and show that $\rho(f)$ and $\sigma(f)$ satisfy the same relations as in complex analysis, with regards to the coefficients a_n. But here another expression $\psi(f)$ that we call cotype of f, depending on the number of zeros inside disks is very important and we show under certain wide hypothesis, that $\psi(f) = \rho(f)\sigma(f)$, a formula that has no equivalent in complex analysis and suggests that it might hold in the general case. We show that $\rho(f) = \rho(f')$, $\sigma(f) = \sigma(f')$ and present an asymptotic relation linking the numbers of zeros inside disks for two functions of same order. That applies to a function and its derivative. We show that the derivative of a transcendental entire function f has infinitely many zeros that are not zeros of f and particularly we show that f' cannot divide f, when the p-adic absolute value of the number of zeros of f inside disks satisfies certain inequality and particularly when f is of finite order.

(Joint work with Abdelbaki Boutabaa and Kamal Boussaf)

Theory of infinitely near points in smooth manifolds: the Fermat functor

Paolo Giordano

The work [Wei] of A. Weil on infinitesimal prolongations of smooth manifolds had a great influence on Differential Geometry, inspiring research threads like Weil functors, Synthetic Differential Geometry, Differential Geometry over general base fields and rings, but also Grothendieck's approach to infinitesimal neighborhoods in algebraic geometry.

We present a new approach to the extension of smooth manifolds with infinitely near points which permits to formalize infinitesimal methods in Differential Geometry and has a clear geometrical meaning. In case of a smooth manifold M this extension can be easily formulated. We firstly have to introduce *little-oh polynomials* as maps $x : \mathbb{R}_{\geq 0} \longrightarrow M$ that can be written as $\phi(x(t)) = r + \sum_{i=1}^{k} \alpha_i \cdot t^{a_i} + o(t)$, where $r, \alpha_i \in \mathbb{R}^n$, $a_i \in \mathbb{R}_{\geq 0}$ and $t \to 0^+$, in some chart (U, ϕ) such that $x(0) \in U$. We can hence introduce an equivalence relation between little-oh polynomials saying that $x \sim y$ iff we can write $\phi(x(t)) = \phi(y(t)) + o(t)$ in some chart (U, ϕ) such that $x(0), y(0) \in U$. The extension of M with infinitely near points is simply the quotient set ${}^\bullet M := M/\sim$. This construction applied to $M = \mathbb{R}$ gives the so-called *ring of Fermat reals* ${}^\bullet\mathbb{R}$, a non-archimedean ring with nilpotent infinitesimals, see e.g. [Gio10,Gio11]. However, this construction can be generalized to any diffeological space $X \in \mathbf{Diff}$, obtaining the *Fermat functor* ${}^\bullet(-) : \mathbf{Diff} \longrightarrow {}^\bullet\boldsymbol{\mathcal{C}}^\infty$ from the category $\mathbf{Diff}$ of diffeological spaces to that of *Fermat spaces* ${}^\bullet\boldsymbol{\mathcal{C}}^\infty$. Since the category $\mathbf{Diff}$ is cartesian closed, complete and co-complete, and embeds the category $\mathbf{Man}$ of smooth manifolds, the whole construction can be applied also to infinite dimensional spaces like the space of all the smooth maps between two smooth manifolds and to spaces with singularities. In this setting we can define a tangent vector as an infinitesimal smooth curve $t : D \longrightarrow X$, where $D = \{h \in {}^\bullet\mathbb{R} \mid h^2 = 0\}$ is the ideal of first order nilpotent infinitesimals; we can see any vector field as an infinitesimal transformation of the space X into itself and we can prove that any vector field has a unique infinitesimal integral curve. For the case ${}^\bullet\boldsymbol{\mathcal{C}}^\infty({}^\bullet M, {}^\bullet N)$ this amount to say that an infinite system of ODE has always a unique infinitesimal solution. Even if the whole construction does not need any background of Mathematical Logic, the study of the preservation properties of the Fermat functor ${}^\bullet(-)$ reveals a surprising strong connection with intuitionistic logic. In fact, this functor preserves product of manifolds, open subspaces, inclusion, inverse images, intersections and unions, intuitionistic negation and implication, intuitionistic quantifiers. Therefore, a full transfer theorem for intuitionistic formulas holds and summarizes the preservation properties of this functor.

[Gio10] P. Giordano, The ring of fermat reals, *Advances in Mathematics* 225, pp. 2050–2075, 2010.

[Gio11] P. Giordano, Fermat-Reyes method in the ring of Fermat reals. *Advances in Mathematics* 228, pp. 862–893, 2011.

[Wei] A. Weil, Th*é*orie des points proches sur les vari*é*t*é*s diff*é*rentiables. *Colloque de Géometrie Différentielle*, Strasbourg, pp. 111–117, 1953.

Exponential laws for spaces of ultrametric differentiable functions and applications

Helge Glöckner

We establish exponential laws for certain spaces of differentiable functions over a valued field K. For example, we show that the topological vector spaces $C^{r,s}(UxV, E)$ and $C^r(U, C^s(V, E))$ are isomorphic if U and V are open subsets of K^n and K^m, respectively, E is a topological K-vector space, and r, s are degrees of differentiability. As a first application, we study the density of locally polynomial functions in spaces of partially differentiable functions over an ultrametric field (thus solving an open problem by E. Nagel), and also global approximations by polynomial functions. As a second application, we obtain a new proof for the characterization of C^r-functions on powers Z_p^n of the p-adic integers in terms of the decay of their Mahler expansions. In both applications, the exponential laws enable simple inductive proofs via a reduction to the one-dimensional, vector-valued case.

Reference: *Exponential laws for ultrametric partially differentiable functions and applications*, p-Adic Numbers Ultram. Anal. Appl. **5** (2013), no. 2, 122–159.

On bivariant K-theory for ultrametric Banach algebras

Nikolay Ivankov

Bivariant K-Theory, also knowns a KK-Theory, was introduced by Gennady Kasparov in early 1980s in an approach to the Novikov conjecture, and proved itself to be a powerful tool in the theory of C*-algebras. The main property of this theory is that it unifies K-Theory and K-Homology into a single general framework, incorporating them as particular cases and interpreting the index map in the form of so-called Kasparov product. Since the middle of 00's, versions of bivariant K-theory for other frameworks, usually called kk-theories were introduced by numerous scientists. Among these versions are the ones for locally convex and discrete algebras.

Basing on the general aaproach developed in the works of Cuntz, Cortiñas, Thom and Ellis, we define a version of (equivariant) bivariant K-theory for ultrametric Banach algebras over the algebra C_p of complex p-adic numbers. Our main hypotheses are the follows: 1) there holds an analogue of Green-Julg theorem for the KK-theory of C*-algebras, relating the equivariant theory with non-equivariant for the action of compact zero-dimensional p-free ultrametric groups; 2) there is a natural isomorphism between the group $kk(C_p, A)$ and the ultrametric version of Weibel's homotopy K-theory, applied for ultrametric Banach algebras. We hope that the theory may find its applications in approaches to the Farrell-Jones conjecture for algebraic K-theory and, possibly, p-adic quantum mechanics.

Non-archimedean Grothendieck and Krein's theorems

Jerzy Kąkol, Albert Kubzdela

Let X be a compact Hausdorff space. Grothendieck proved that a bounded set H in the Banach space of real-valued continuous functions $C(X, \mathbb{R})$ is relatively compact in the pointwise topology τ_p if and only if it is relatively compact in the weak topology of $C(X, \mathbb{R})$. This result fails for the space $C(X, \mathbb{K})$ of continuous maps with values in a locally compact non-trivially valued non-Archimedean field $\mathbb{K}$. However, we prove that if X is an infinite zero-dimensional compact space and $H \subset C(X, \mathbb{K})$ is uniformly bounded, then the absolutely convex hull $acoH$ is τ_p-relatively compact if and only if H is $w-$relatively compact.

The classical Krein's theorem states that in a real or complex Banach space E the convex hull coM is weakly relatively compact for a weakly relatively compact set $M \subset E$. This fact can be deduce from quantitative versions of this theorem which use measures of weak noncompactness. We introduce a few measures of weak noncompactness, defined on non-Archimedean Banach spaces over a locally compact valued field and compare their properties with the classical counterparts. Finally, we prove non-Archimedean, quantitative versions of Krein's theorem.

Large affine spaces of invertible operators

Hans A. Keller

Let E be a Banach space over a non-archimedianly valued field $(K, |\,.\,|)$. Consider the algebra $\mathcal{B}(E)$ of bounded linear operators $B : E \to E$ and the subgroup $\mathcal{G}(E)$ of invertible operators. Then $\mathcal{G}(E) \cup \{0\}$ is a cone in the vector space $\mathcal{B}(E)$. From the geometric point of view there arise several natural questions, in particular the following.

(1) What are the maximal linear subspaces of $\mathcal{B}$ completely contained in $\mathcal{G} \cup \{0\}$?

(2) What are the maximal linear subspaces of $\mathcal{B}$ contained in $\mathcal{B} \setminus \mathcal{G}$?

(3) What are the maximal *affine* subspaces of $\mathcal{B}$ contained in $\mathcal{G} \cup \{0\}$?

(2) is easily settled but (1) and (3) are intricate even in finite dimension because the answer depends on the arithmetic of the base field. We shall examine problems (1) and (3) in the case of the non-archimedian orthomodular $(E, \langle\,.,.\,\rangle)$ described in previous papers. In our talk we shall first summarize the construction of E and then provide answers to the above problems. The base field K has a valuation of infinite rank, and to the space E there belongs an infinite sequence of residual spaces $\hat{E}_0, \hat{E}_1, \hat{E}_2, \ldots$. Again the technique of reduction to the residual spaces turns out to be a most powerful tool.

p-adic numbers from superstrings and molecular motors to cognition and psychology

Andrei Khrennikov

We start with a brief mathematical introduction in theory of p-adic numbers and corresponding analysis (geometrically this is analysis on homogeneous trees). Then we again briefly present applications to physics, supestring theory, quantum mechanics, and the theory of disordered systems. The next part of the talk is devoted to applications to genetics and cellular biology, in particular, we plan to present a mathematical model of functioning of molecular motors in cells. The last part is devoted to application of theory of p-adic dynamical systems to cognition and psychology with the final arrival to the p-adic dynamical model for Freud's psychoanalysis.

References:

1. A. Khrennikov, Information dynamics in cognitive, psychological, social, and anomalous phenomena. Ser.: Fundamental Theories of Physics, Kluwer, Dordreht, 2004.

2. V. Anashin and A. Khrennikov, Applied algebraic dynamics. De Gruyter, Berlin, 2009.

3. A.Yu. Khrennikov, S.V. Kozyrev, A. Mansson, Hierarchical model of the actomyosin molecular motor based on ultrametric diffusion with drift. `arXiv:1312.7528 [q-bio.BM]`.

4. A.Yu. Khrennikov, S.V. Kozyrev, p-Adic numbers in bioinformatics: from genetic code to PAM-matrix. Journal of Theoretical Biology. 2009. V.261. P.396–406; `arXiv:0903.0137 [q-bio.GN]`

Ball spaces - a general framework for fixed point theorems, I

Katarzyna Kuhlmann

A theorem of considerable importance in valuation theory is the ultrametric version of Banach's Fixed Point Theorem (FPT) which was first proved by S. Priess-Crampe. This version requires an analogue of completeness which is called "spherical completeness". These and related ultrametric theorems constitute underlying principles for important results and tools in valuation theory, reducing their proofs to the essential. Driving this abstraction even further, one can ask for the common denominator between metric and ultrametric FPTs and the connections to topological methods. We present an abstract but simple and efficient approach to such a unification of methods in Fixed Point Theory. Inspired by the ultrametric world, we base this approach on the notion of spherical completeness. The key idea is to let it refer to any collection of distinguished subsets of an arbitrary set, instead of just the ultrametric balls in an ultrametric space. It turns out that important completeness properties can be formulated in terms of spherical completeness. For example, a topological space is compact if and only if it is spherically complete with respect to its nonempty closed sets.

We present general FPTs which then can be specialized to FPTs in several different situations by choosing the "balls" (the distinguished sets) according to the application we have in mind. This way we obtain new, along with well established, FPTs. Having a common denominator for the various applications also allows us to transfer known results from one application to another where they had not been previously observed. Examples for such transfers are the Knaster-Tarski Theorem from the theory of complete lattices and the Tychonoff Theorem for products of compact topological spaces.

(Joint work with Franz-Viktor Kuhlmann)

Ball spaces - a general framework for fixed point theorems, II

Franz-Viktor Kuhlmann

In this talk we discuss the specifically ultrametric applications of the general theorems presented in the first talk. We present an Attractor Theorem that has been used to prove generalizations of Hensel's Lemma as well as multi-dimensional and even infinite-dimensional Implicit Function Theorems. Generalizations of this Attractor Theorem lead to a Common Point Theorem and other theorems, which we will compare with Common Point Theorems and Multivalued Fixed Point Theorems which were proved by Priess-Crampe and Ribenboim.

(Joint work with Katarzyna Kuhlmann)

A nonarchimedean counterpart of Johnson's theorem for discrete groups

Yuri Kuzmenko

The concept of amenability for a Banach algebra arises naturally in cohomology theory for Banach algebras. It was introduced by B.E. Johnson in 1972 and has proved to be of enormous importance in Banach algebra theory over $\mathbb{C}$. In particular, Johnson showed that a locally compact group is amenable if and only if the algebra $L^1(G)$ is amenable.

The concept of K-amenability for a locally compact group where K is a nonarchimedean valued field was introduced by W.H. Schikhof in 1975. It is natural to ask if there is a counterpart of Johnson's theorem in nonarchimedean case. For example, it is easy to see, that the group $\mathbb{Z}/p\mathbb{Z}$ is not $\mathbb{Q}_p$-amenable, but the algebra $l^1(Z/pZ)$ is $\mathbb{Q}_p$-amenable.

In this talk we define a notion of Johnson amenable group, by weakening one of the conditions in Schikhof's definiton. The main result is that in the case of a spherically complete field K the discrete group G is Johnson amenable if and only if the K-algebra $l^1(G)$ is amenable.

p-adic fractal strings: fractal tube formulas, zeta functions and complex dimensions

Michel L. Lapidus

We present an explicit volume formula for the tubular neighborhoods of a p-adic fractal string, expressed in terms of the underlying complex dimensions (i.e., the poles of an associated geometric zeta function). The resulting general fractal tube formula is illustrated by some simple examples, including the nonarchimedean Cantor, Euler and Fibonacci strings, and more generally, by the important special case of p-adic self-similar fractal strings. This work is joint with Hung Lu and Machiel van Frankenhuijsen, and relies on earlier work on p-adic fractal strings by the author with Hung Lu, as well as on the general theory of complex dimensions of archimedean (or real) fractal strings developed by the author and Machiel van Frankenhuijsen presented in the second revised and enlarged edition of the book (by MLL and M-F) "Fractal Geometry, Complex Dimensions and Zeta Functions: Geometry and Spectra of Fractal Strings" (Springer Monographs in Mathematics, Springer, New York, 2013, approx. 600 pages). If time permits, several open problems will be proposed, connected with aspects of p-adic harmonic analysis and complex analysis, as well as with number theory and the possibility of extending the theory to the adelic setting and to the setting of (nonarchimedean) Berkovich spaces.

J-stability of immediately expanding polynomial maps in non-Archimedean dynamics

Junghun Lee

In the theory of dynamical system, one of the assignments is about the stability of the dynamical systems, which states that if two given dynamical systems are "close enough, then there exists a conjugation between those two dynamical systems. Recently, I have succeeded to prove the stability of the Julia set, which is a chaotic locus of a given dynamical system, of some polynomial maps in non-Archimedean dynamical systems as in complex dynamical systems by using the non-Archimedean properties.

Some properties of Liouville numbers in the non-archimedean case

Hamza Menken

In the present work, we investigate some properties of Liouville numbers in the non-Archimedian case. As non-Archimedian case we consider the field of p-adic numbers $\mathbb{Q}_p$ and the functions field $K < x >$ where K is an arbitrary field.

(Joint work with Abdulkadir Aşan).

References

1. Mahler, K., On a theorem of Liouville in fields of positive characteristic, Canad. J. Math. 1 (1949), 397-400.
2. Hancl, J. Liouville sequences. Nagoya Math. J. , Vol. 172 (2003) 173-187.
3. Chaichana, T., Komatso, T. and Laohakosol, V. Liouville numbers in the non-archimedean case. Publ. Math. Debrecen 77 (2010), no. 1-2 , 39–63.

p-adic Taylor polynomials

Enno Nagel

Checking for non-Archimedean differentiability by definition, through partial difference quotients, is difficult, especially in higher degrees. We review when an equivalent easier criterion, through Taylor polynomials, comes to rescue, and compare it with that used in a recent definition of differentiability from Representation Theory.

p-adic differential equations on curves

Andrea Pulita

Recent developments on p-adic differential equations permit to establish necessary and sufficient criteria in order to have finite dimensional de Rham cohomology. We give an overview of these results (This is a joint work with J.Poineau). We then focus on a result about cyclic vector, and about Frobenius.

Fundamental solutions of pseudo-differential equations associated with quadratic forms

John Jaime Rodríguez

Let $S(\mathbb{Q}_p)$ be the space of test functions on $\mathbb{Q}_p$, $(\mathcal{F}\phi)(\xi)$ be the Fourier transform of the function $\phi(x) \in S(\mathbb{Q}_p)$ and $(.,.)_p$ the Hilbert symbol. For a quadratic form $f(\xi) = a_1\xi_1^2 + a_2\xi_2^2 + \cdots + a_n\xi_n^2$ let $D = a_1 \cdots a_n$ and $D^* = (-1)^{n/2}D$.

Consider the twisted pseudo-differential operator defined by

$$(\mathbf{f}(\partial, \alpha, \pi_{D^*})\phi)(x) = \mathcal{F}^{-1}_{\xi\to x}\Big(\pi_{D^*}(f^*(\xi)|f^*(\xi)|_p^{\alpha}\mathcal{F}_{x\to\xi}\phi\Big),$$

with $f(\xi) = a_1\xi_1^2 + a_2\xi_2^2 + \cdots + a_n\xi_n^2$, $a_i \in \mathbb{Q}_p^{\times}$, $f^*(\xi) = f(\frac{\xi_1}{a_1}, \ldots, \frac{\xi_n}{a_n})$ and $\pi_{D^*}(t) = (D^*, t)_p$.

In this talk we find the fundamental solution associated to this operator in the case n even and $n > 4$.

The method we use consits in finding a functional equation for the local zeta function

$$Z_\phi(s, \pi_\beta, f) = \int_{\mathbb{Q}_p^n \setminus f^{-1}(0)} \pi_\beta(f(x))|f(x)|_p^{s-n/2}\phi(x)d^nx,$$

and use its meromorphic continuation.

The infinity computer and numerical computations with infinities and infinitesimals

Yaroslav D. Sergeyev

The lecture introduces a new methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers on a new type of a computer, the Infinity Computer (see EU, USA, and Russian patents). The new approach is based on the principle “The part is less than the whole introduced by Ancient Greeks that is applied to all numbers (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework different from that of the non-standard analysis.

The new methodology evolves ideas of Cantor in a more applied way and introduces new infinite integers that possess both cardinal and ordinal properties as usual finite numbers. It gives the possibility to execute computations of a new type and simplifies fields of mathematics where the usage of the infinity and/or infinitesimals is necessary (e.g., divergent series, limits, derivatives, integrals, measure theory, probability theory, fractals, etc.). Numerous examples and applications are given. A number of results related to the First Hilbert Problem and Riemann zeta function are established.

In the following there are listed both operations that the Infinity Computer can execute and traditional computers are not able to perform and some of new areas of applications. The new approach allows:

- to substitute symbols + and − by sets of positive and negative infinite numbers, to represent them in the memory of the Infinity Computer and to execute arithmetical operations with all of them numerically, as we are used to do with usual finite numbers on traditional computers;
- to substitute qualitative descriptions of the type "a number tends to zero by precise infinitesimal numbers, to represent them in the memory of the Infinity Computer, and to execute arithmetical operations with them numerically as we are used to do with usual finite numbers using traditional computers;
- to calculate divergent limits, series, and improper integrals, providing as results explicitly written different infinite numbers, to be possibly used in further calculations on the Infinity Computer;
- to avoid appearance of indeterminate forms (e.g., in situations where it becomes necessary to calculate difference of two divergent series);
- to evaluate functions and their derivatives at infinitesimal, finite, and infinite points (infinite and infinitesimal values of functions and their derivatives can be also explicitly calculated);
- to study divergent processes at different infinite points;
- to introduce notions of lengths, areas, and volumes of fractal objects obtained after infinite numbers of steps and compatible with traditional lengths, areas, and volumes of non-fractal objects and to calculate all of them in a unique framework.

The Infinity Calculator using the Infinity Computer technology is presented during the talk. Additional information can be downloaded from the page `http://www.theinfinitycomputer.com`

New results on the Lebesgue-like measure and integration theory on the Levi-Civita field and applications

Khodr Shamseddine

In this talk, we will generalize the Lebesgue-like measure and integration theory on the Levi-Civita field to two and three dimensions, showing that the resulting measures and double and triple integrals have similar properties to those from Real Analysis. Then we will introduce so-called delta functions which are piecewise analytic and integrable on the whole space with integral equal to 1 and which reduce to the Dirac delta function when restricted to real points. Finally we will present simple applications of the theory.

(Joint work with Darren Flynn)

Superlinear convergent iterative methods on non-archimedean fields and their applications

Alexander Wittig

The use of iterative methods on the Levi-Civita field $\mathcal{R}$ and similar non-Archimedean structures is a well established technique in both theoretical study of the properties of $\mathcal{R}$ as well as in practical computations on $\mathcal{R}$. Of particular use in this area is the fixed point theorem for contractions with infinitely small contraction factor, which allows the iterative construction of the fixed point in a mode in which the valuation of the error decreases linearly.

We introduce a superlinearly convergent iterative fixed point method on $\mathcal{R}$ for which the error between the fixed point and the iterates instead decreases quadratically or better in valuation. To easily derive a large class of these super linearly convergent operators, we introduce the Newton method from classical numerical analysis on $\mathcal{R}$.

Examples of superlinearly convergent operators derived both manually as well as using the Newton method are provided for the calculation of the multiplicative inverse, matrix inversion as well as the square root on $\mathcal{R}$ and other generalized Levi-Civita algebras.

We then proceed to apply the concept of the superlinear convergent methods to the intermediate value theorem for power series over $\mathcal{R}$. By judiciously adjusting the fixed point problem being considered in each step of the iteration based on the result of the previous step, this results in a more compact and streamlined proof than currently available ones.

We conclude by applying some of the superlinearly convergent iterations to a computer implementation of the partially ordered algebras ${}_nD_v$. In particular, algorithms as well as implementations of the super linearly convergent methods are shown and compared to the linear methods currently implemented in the COSY INFINITY computation environment, which illustrates the superior numerical performance of the super linearly convergent methods.

(Joint work with Martin Berz)

On complemented subspaces of non-archimedean generalized power series spaces

Agnieszka Ziemkowska

One of the most important problems in the theory of non-archimedean Fréchet spaces is the following one: Let E be a Fréchet space with a Schauder basis. Does every complemented subspace F of E have a Schauder basis? In our paper we prove that for some non-archimedean generalized power series spaces $D_f(a, r)$ this problem has a positive answer. In our previous paper (published in Canadian Journal of Mathematics) we proved a similar result for non-archimedean power series spaces.

(Joint work with Wiesław Śliwa).

Nonlocal operators, parabolic-type equations, and ultrametric random walks

W. A. Zúñiga-Galindo

During the last twenty-five years there have been a strong interest on random walks on ultrametric spaces mainly due its connections with models of complex systems, such as glasses and proteins. Random walks on ultrametric spaces are very convenient for describing phenomena whose space of states display a hierarchical structure. In the middle of the eighties G. Frauenfelder, G. Parisi, D. Stain, among others, proposed using ultrametric spaces to describe the states of complex biological systems, which possess a natural hierarchical organization. Avetisov et al. have constructed a wide variety of models of ultrametric diffusion constrained by hierarchical energy landscapes. From a mathematical point view, in these models the time-evolution of a complex system is described by a p-adic master equation (a parabolic-type pseudodifferential equation) which controls the time-evolution of a transition function of a random walk on an ultrametric space, and the random walk describes the dynamics of the system in the space of configurational states which is approximated by an ultrametric space ($\mathbb{Q}_p$).

We will report on the results of [1, 2]. In [1], we introduced a new class of nonlocal operators which includes the Vladimirov operator. These operators are determined by a radial function which determines the structure of the energy landscape being used. We study a large class of solvable models, we have called them polynomial and exponential landscapes, which includes the linear and exponential landscapes considered by Avetisov et al. We attach to each of these operators a Markov process (random walk) which is bounded and has do discontinuities other than jumps. We solve the Cauchy problem for the master equations attached to these operators and study the first passage time problem for the random walks attached to polynomial landscapes.

In [2], we introduce a new class of parabolic-type pseudodifferential equations with variable coefficients, which contains the one-dimensional p-adic heat equation, the equations studied by Kochubei, and the equations studied by Rodrguez-Vega. We establish the existence and uniqueness of solutions for the Cauchy problem for such equations. We show that the fundamental solutions of these equations are transition density functions of Markov processes.

1. Chacón-Cortes L. F., Zúñiga-Galindo W. A., Nonlocal Operators, Parabolic-type Equations, and Ultrametric Random Walks, J. Math. Phys. 54, 113503 (2013). `arXiv:1308.5013`

2. Chacón-Cortes L. F., Zúñiga-Galindo W. A., Non-local Operators, Non-Archimedean Parabolic-type Equations with Variable Coefficients and Markov Processes, `arXiv:1405.3016`.

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