BOOK OF ABSTRACTS

for the international workshop on

New Trends in Probabilistic Number Theory and the Theory of Stochastic Processes

PADERBORN UNIVERSITY
The University for the Information Society

PADERBORN, JUNE 28-29, 2019

Organizing committee:
Karl-Heinz Indlekofer, Attila Gilányi, Imre Kátai, Oleg I. Klesov, Martin Kolb,
Eugenijus Manstavicius, Thomas Richthammer, Josef G. Steinebach

PROGRAM OF THE WORKSHOP

FRIDAY, 28.06.2019
ROOM L3.204

08:50-09:00	*Opening*
09:00-09:30	Karl-Heinz Indlekofer
09:30-10:00	Allan Gut and Ulrich Stadtmüller
10:00-10:30	Gintautas Bareikis and Algirdas Mačiulis
10:30-11:00	*Coffee break*
11:00-11:30	Allan Gut
11:30-12:00	Vilius Stakėnas
12:00-12:30	Maryna Ilienko
12:30-14:00	*Lunch. Mensa Academica*
14:00-14:30	Oleg I. Klesov
14:30-15:00	Eugenijus Manstavičius
15:00-15:30	Andrii Ilienko
15:30-16:00	*Coffee break*
16:00-16:30	Gediminas Stepanauskas
16:30-17:00	Attila Gilányi
17:00-18:00	*Rest*
18:00-19:00	*Organ concert. Paderborner Dom*
19:00-21:00	*Welcome party. Restaurant Phoenix*

SATURDAY, 29.06.2019
ROOM L3.204

09:30-10:00	Gintautas Bareikis and Algirdas Mačiulis
10:00-10:30	Vytautas Stepas
10:30-11:00	Alexander V. Ivanov, Igor V. Orlovskyi
11:00-11:30	Josef G. Steinebach
11:30-11:35	*Closing*
11:35-12:00	*Transfer to Heinz Nixdorf MuseumsForum*
12:00-14:00	*Lunch at Heinz Nixdorf MuseumsForum*
14:00-15:00	*Guided excursion in Heinz Nixdorf MuseumsForum*

Abstracts

About arithmetical processes related to the natural divisors

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Abstract

For an arithmetic multiplicative function $f \geq 0$ the generalized divisor function $\mathcal{T}(m,v) := \sum_{d|m,\ d\leq v} f(d),\ m \in \mathbb{N},\ v \in \mathbb{R},$ and corresponding arithmetically defined random process $X(m,t) := \mathcal{T}(m,m^t)/\mathcal{T}(m,m)$, $0 \leq t \leq 1$, are considered.

Investigations of linear functional equations with computer

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Abstract

Linear functional equations have several applications in probability theory. In this talk, we present a computer program package developed in the computer algebra system Maple for investigating such types of functional equations. Besides the consideration of general properties of the package, we describe some of its new components, which can be used to examine the so-called alienness property of functional equations.

Two examples on how mathematics can help to understand the world

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Abstract

In this talk I will present two aspects of human life and "explain" how the language of mathematics and probability, in a beautiful manner, can make them clear(er) and (more) reasonable.

On the convergence of point processes associated with coupon collector's and Dixie cup problems

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Abstract

The coupon collector's problem (CCP) as well as its generalization known as the Dixie cup problem (DCP) belong to the classics of combinatorial probability. Their statements are as follows: a person collects coupons, each of which belongs to one of n different types. The coupons arrive one by one at discrete times, the type of each coupon being equiprobable and independent of types of preceding ones. Let T_n^r stand for the (random) number of coupons a person needs to collect in order to assemble $r \in \mathbb{N}$ complete collections. The most typical questions concern asymptotics of $\mathbb{E}T_n^r$ and distributional limit theorems for T_n^r themselves as $n \to \infty$. The case $r = 1$ refers to CCP while $r \geq 2$ to DCP.

CCP, DCP and their further generalizations have a long history going back to de Moivre, Euler and Laplace. In particular, recall a classic result by Erdős and Rényi (1961):

$$\mathbb{E}T_n^r = n \ln n + (r-1)n \ln\ln n + (\gamma - \ln(r-1)!)n + o(n), \tag{1}$$

$$\lim_{n\to\infty} \mathbb{P}\Big\{\frac{T_n^r}{n} - \ln n - (r-1)\ln\ln n < x\Big\} = \exp\Big\{-\frac{e^{-x}}{(r-1)!}\Big\}, \tag{2}$$

with $\gamma = -\Gamma'(1)$ standing for the Euler–Mascheroni constant.

In his seminal paper, Holst (1986) proposed a fruitful poissonization idea which allowed to prove limit results like (1), (2) avoiding intricate combinatorial calculations. In a very recent paper by Glavaš and Mladenović (2018), the connections between CCP and Poisson processes were shown to be even more tight. As a matter of fact, it was proved that the point processes given by the times of first arrivals for coupons of each type, centered and normalized in a proper way, converge toward a non-homogeneous Poisson point process as $n \to \infty$. The above convergence is, as usual, understood as the distributional one in the space of all locally finite point measures endowed with the vague topology.

Inspired by this paper, we give a generalization of the above result for DCP. The latter, in turn, allows to obtain non-trivial convergence results for some other interesting characteristics in DCP. Our proof differs from that by Glavaš and Mladenović, involving poissonization technique in the spirit of Holst and some depoissonization tricks instead of combinatorial arguments.

On the convergence of series $\sum_{n=1}^{\infty} a_n S_n$ for autoregression sequences

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Abstract

Given a sequence of independent r.v.'s (X_k), asymptotic behaviour of sums $S_n = \sum_{k=1}^{n} X_k$, as well as convergence a.s. of different types of series are of great importance in probability theory. In particular, some authors (see, [1, 2]) were attracted by the problem of convergence/divergence a.s. of the series $\sum_{n=1}^{\infty} a_n S_n$, with $a_n = 1/n^{\alpha}$, $n \geq 1$, as the most popular weights. In [3] the necessary and sufficient conditions for the convergence a.s. of the series $\sum_{n=1}^{\infty} |S_n|/n^{\alpha}$ are the subject of interest.

Following the lines of mentioned papers, we study similar problems for autoregression sequences. Namely, consider a zero-mean linear autoregression model $(\xi_k, \quad k \geq 1)$:

$$\xi_1 = \eta_1, \quad \xi_k = \alpha_k \xi_{k-1} + \eta_k, \quad k \geq 2, \tag{3}$$

where (α_k) is a nonrandom real sequence, and (η_k) is a sequence of independent symmetric random variables. Set $S_n = \sum_{k=1}^{n} \xi_k$, $n \geq 1$, and study convergence a.s. of the series

$$\sum_{n=1}^{\infty} \frac{S_n}{n^{\alpha}}.$$

Our main results, see [4, 5], naturally generalize those in the independent case.

References

[1] Koopmans L.H., Martin N., Pathak P.K., C., *On the divergence of a certain random series* // Ann. of Probab. — Vol. 2, No. 3. — 1974. — P. 546-550.

[2] Gaposhkin V.F., *Necessary convergence conditions for series* $\sum a_n S_n$ *in the case of identically distributed independent random quantities* // Math. Notes. — Vol. 20, No. 4. — 1976. — P. 852-857.

[3] Deli Li, Yongcheng Qi, Rosalsky A., *A refinement of the Kolmogorov-Marcinkiewicz-Zygmund strong law of large numbers*// J. Theor. Probab. — 2011. — Vol. 24. — P. 1130–1156.

[4] Buldygin V., Runovska M. (Ilienko M.) *Sums whose terms are elements of linear random regression sequences.* — LAP, 2014. — 168 p.

[5] Ilienko M., *A note on the Kolmogorov-Marcinkiewicz-Zygmund type strong law of large numbers for elements of autoregression sequences*// Theory Stoch. Process. — 2017. — Vol. 22(38), No. 1. — P. 22–29.

On the uniform distribution and the uniform summability of multiplicative functions

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June 25, 2019

Abstract

In [1] we introduced the space $\mathcal{L}^*$ of *uniformly summable functions* $f : \mathbb{N} \to \mathbb{C}$. Here $f \in \mathcal{L}^*$ iff

$$\overline{\lim_{N\to\infty}} N^{-1} \sum_{n\le N} |f(n)| < \infty \quad \text{and} \quad \lim_{K\to\infty} \sup_{N\ge 1} N^{-1} \sum_{\substack{n\le N \\ |f(n)|\ge K}} |f(n)| = 0.$$

The idea of uniform summability turned out to provide the appropriate tools for dealing with the mean behaviour of multiplicative and additive functions (cf. [3]).

In [2] we described the connections of uniform summability with the existence of a limit distribution for *real-valued* multiplicative functions and the uniform distribution of *positive valued* multiplicative functions. Following Diamond and Erdös [1] we say that the values of a function $f : \mathbb{N} \to (0,\infty)$ are *uniformly distributed* in $(0,\infty)$ (briefly, f is u.d. in $(0,\infty)$) if $f(n)$ tends to infinity as $n \to \infty$ and if there exists a positive c such that

$$N(y,f) := \sum_{\substack{n \\ f(n)\le y}} 1 \sim cy \quad as \quad y \to \infty.$$

In this talk we present new results [4] obtained in the frame of the DFG project "Ein einheitlicher Zugang zu Grenzwertsätzen für duale Objekte in Wahrscheinlichkeits- und Zahlentheorie".

References

[1] H. Diamond and P. Erdös, Multiplicative functions whose values are uniformly distributed in $(0,\infty)$, In: Proc. Queen's Number Theory, 1979, (ed. P. Ribenboim), Queen's Papers in Pure and Appl. Math., Queen's Univ. Kingston, Ont, (1980), pp. 329-378.

[2] K.-H. Indlekofer, *A mean-value theorem of multiplcative arithmetical functions*, Math. Z. **172**, 1980, 255–271.

[3] K.-H. Indlekofer, *Limiting distributions and mean-values of multiplicative arithmetical functions*, J. Reine Angew. Math., **328**, 1981, 116–127.

[4] K.-H. Indlekofer, *On the uniform distribution and uniform summability of positive valued multiplicative functions. (Preprint)*

[5] E. Kaya and R. Wagner, *On some results of Indlekofer for multiplicative functions*, Annales Univ. Sci. Budapest., Sect., Comp. **48**, 2018, 17–29.

On regularly varying additive and multiplicative functions

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Abstract

The function
$$f(n) = c \log n$$
is called a *regular additive*, while
$$g(n) = \exp(s \log n), \qquad s = a + it,$$
is called a *regular multiplicative function.* These functions have some interesting properties which characterize them. Klurman and Mangarel proved a nice theorem using an idea of Terence Tao.

Law of the iterated logarithm for inverse subordinators

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Abstract

The law of the iterated logarithm is proved for inverse subordinators. The limsup is nonrandom in contrast to what is claimed by some other authors. An application to some risk models is discussed.

We study the asymptotic behavior of inverse subordinators constructed from various types of Lévy processes. A particular case is presented by inverse stable subordinats that are used in a probability model for time-fractional differential equations. A number of applications of inverse stable subordinats is known in variety of problems in mathematics and physics. Several different governing equations for the inverse stable subordinator have been proposed in the literature and the almost sure limit behavior of a solution of each of the equations has its own limit dynamics.

Many modern models are based on some stochastic process delayed by inverse subordinators. The limit behavior of a delayed process is determined by that of an underlying inverse subordinator, however it may essentially be different.

Note that the change of time may be quite complicated and this also changes the asymptotics.

Some examples and subordinators of inverse subordinators are discussed by Bertoin in [2].

The proofs use general ideas of limit theorems of generalized inverse functions discussed in [1] in detail.

This talk is based on joint work (in progress) with Josef Steinebach (Universität zu Köln) and Nikolaiĭ Leonenko (Cardiff University, Great Britain).

Reference

[1] V. V. Buldygin, K.-H. Indlekofer, O. I. Klesov, J.G. Steinebach (2018). Pseudo-Regularly Varying Functions and Generalized Renewal Processes, Springer, Cham, Switzerland.

[2] J. Bertoin, Subordinators: examples and applications, Ecole d’été de Probabilitiés de St-Flour, 1997.

Modeling the Dirichlet distribution using multiplicative functions

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Abstract

Let a, b, c be positive constants and

$$E(u,v) := \{(s,t) \mid 0 \le s \le u,\ 0 \le t \le v,\ s+t \le 1\}.$$

The two dimensional Dirichlet distribution concentrated on the triangle $E(1,1)$ is defined by the distribution function

$$D(u,v;a,b,c) = \frac{\Gamma(a+b+c)}{\Gamma(a)\Gamma(b)\Gamma(c)} \iint\limits_{E(u,v)} \frac{dt\,ds}{t^{1-a}s^{1-b}(1-t-s)^{1-c}}.$$

We prove that some class of Dirichlet distributions can be simulated by means of a sequence of distributions defined via multiplicative functions related to the generalized divisors function.

Optimal constants in some conditional variance estimates

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Abstract

We deal with the ubiquitous Ewens distribution defined on the set of vectors $\omega = (\omega_1, \ldots, \omega_n) \in \mathbf{N}_0^n$ by

$$P_\theta(\{\omega\}) = \frac{\mathbf{1}\{\ell(\omega) = n\}}{\Theta(n)} \prod_{j=1}^{n} \Big(\frac{\theta}{j}\Big)^{\omega_j} \frac{1}{\omega_j!}.$$

Here $\theta > 0$ is a parameter, $\Theta(k) = \theta(\theta+1)\cdots(\theta+k-1)/k!$, $\mathbf{1}\{\cdot\}$ stands for an indicator, and $\ell(\omega) := 1\omega_1 + \cdots + n\omega_n$. It is just the conditional distribution, namely,

$$P_\theta(\{\omega\}) = P\big(\xi = \omega \big| \, \ell(\xi) = n\big),$$

where $\xi = (\xi_1, \ldots, \xi_n)$ is a vector of independent Poisson coordinates with $\mathbf{E}\xi_j = \theta/j$ if $1 \le j \le n$.

The problem is to estimate the variance Var_θ with respect to P_θ of the linear statistics $h(\omega) = a_1\omega_1 + \cdots + a_n\omega_n$ via the sum of variances of these dependent summands. The main result is the following sharp inequality

$$\mathrm{Var}_\theta h \le \frac{\theta(\theta+2)}{\theta+1} \sum_{j=1}^{n} \frac{a_j^2}{j} \frac{\Theta(n-j)}{\Theta(n)}, \quad n \ge 2.$$

The search of extremal $a_1, \ldots, a_n$ is built upon the discrete Hahn's polynomials. Applied for weighted random permutations, the result gives an analog of the Turán-Kubilius inequality with the exact constant. The details are exposed in the following joint papers:

1. J. Klimavičius and E. Manstavičius, The Turán–Kubilius inequality on permutations, *Annales Univ. Sci. Budapest., Sect. Comp.* **48** (2018), 45–51.

2. Z. Baronėnas, E. Manstavičius and P. Šapokaitė, A sharp inequality for the variance with respect to the Ewens Sampling Formula (submitted).

Large deviations of the least squares estimator in continuous-time models with sub-Gaussian noise

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Consider a regression model

$$X(t) = a(t,\, \theta) + \varepsilon(t),\ t \geq 0,$$

where $a(t,\, \tau)$, $(t,\, \tau) \in \mathbb{R}_+ \times \Theta^c$, is a continuous function, true parameter value $\theta = (\theta_1, ..., \theta_q)'$ belongs to an open bounded convex set $\Theta \subset \mathbb{R}^q$ and random noise $\varepsilon = \{\varepsilon(t),\ t \in \mathbb{R}\}$ satisfies the condition **N** below.

Definition 1. Random vector $\xi = (\xi_1,\ \ldots,\ \xi_n)' \in \mathbb{R}^n$ is called strictly sub-Gaussian if for any $\Delta = (\Delta_1,\ \ldots,\ \Delta_n)' \in \mathbb{R}^n$

$$E \exp\left\{ \sum_{i=1}^{n} \xi_i \Delta_i \right\} \leq \exp\left\{ \frac{1}{2} \sum_{i,j=1}^{n} B(i,j) \Delta_i \Delta_j \right\},$$

where $B(i,j) = E\xi_i \xi_j$, $i,j = \overline{1,n}$.

Definition 2. $\{\xi(t),\ t \in \mathbb{R}\}$ is said to be jointly strictly sub-Gaussian stochastic process, if for any $n \geq 1$, and any $t_1,\ \ldots,\ t_n \in \mathbb{R}$ random vector $\xi_n = \Big(\xi(t_1),\ \ldots,\ \xi(t_n)\Big)'$ is strictly sub-Gaussian.

N. ε is a mean square and almost sure continuous jointly strictly sub-Gaussian stochastic process, $E\varepsilon(t) = 0$, $t \in \mathbb{R}$.

Definition 3. Any random vector $\theta_T = (\theta_{1T},\ \ldots,\ \theta_{qT})' \in \Theta^c$ having the property

$$Q_T(\theta_T) = \inf_{\tau \in \Theta^c} Q_T(\tau),\ Q_T(\tau) = \int \Big[X(t) - a(t,\, \tau)\Big]^2 dt.$$

is said to be the least squares estomator of unknown parameter θ obtained by the observations $\big\{X(t),\ t \in [0,T]\big\}$.

Upper exponential bounds for probabilities of large deviations of the least squares estimator of nonlinear regression parameter in discrete-time models with jointly strictly sub-Gaussian random noise were obtained in Ivanov [1]. Statements presented in the talk extend some results of [1] to continuous-time observation models. There are also some examples of regression function and random noise, that satisfy conditions of the obtained theorems.

References

[1] Ivanov, A.V. *Large deviations of regression parameter estimate in the models with stationary sub-gaussian noise* // Theor. Probability and Math. Statist. – 2017. – Vol. 95. – P. 99–108.

Variations of the elephant random walk

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Abstract

In the classical simple random walk the steps are independent, viz., the walker has no memory. In contrast, in the Elephant Random Walk (ERW) which was introduced by Schütz and Trimper in 2004, the walker remembers the whole past, and the next step always depends on the whole path so far. Sevaral papers have appeared for the ERW mainly from physicists, the most relevant mathematical paper is Bercu (2018). Our main aim is to prove results when the elephant has only a restricted memory, for example remembering only the most remote step(s), the most recent step(s) or both. We also extend the models to cover more general step sizes and walks with delays.

Cantor expansions, rational numbers and arithmetical functions

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Abstract

Let $\{b_n\}_{n\geqslant 1}, b_n \geqslant 2$, be a sequence of integers, $B_1 = b_1, B_{n+1} = b_{n+1}B_n$, as $n \geqslant 1$. For $\alpha \in (0;1]$ consider the expansion in base $\{B_n\}$ (Cantor expansion):

$$\alpha = \sum_{i=1}^{\infty} \frac{d_i}{B_i}, \quad 0 \leqslant d_i < b_i. \tag{4}$$

For the uniqueness property we require that for every n there should exist $m > n$ such that $d_m < b_m - 1$. If $d_n > 0$ and $d_m = 0$ for all $m > n$, we say that the length of expansion (4) is n.

Let $\mathbb{Q}_n$ be the subset of numbers $\alpha \in (0;1]$ having the expansion (4) of length at most n. Then $\mathbb{Q}_1 \subset \mathbb{Q}_2 \subset \cdots \subset \mathbb{Q}$, where by $\mathbb{Q}$ we denote the set of rational numbers $r, 0 < r \leqslant 1$. Obviously, if for any natural number m there exists B_k, such that $m|B_k$, then $\cup_{n=1}^{\infty}\mathbb{Q}_n = \mathbb{Q}\backslash\{1\}$.

Let $f : \mathbb{Q} \to \mathbb{R}$ be some function. We are interested in the value distribution of f in respect to the sequence of sets $\mathbb{Q}_1 \subset \mathbb{Q}_2 \subset \cdots \subset \mathbb{Q}$, i.e., in the behaviour of frequencies

$$|\{r \in \mathbb{Q}_n : f(r) \in S_n\}|/|\mathbb{Q}_n|, \quad n \to \infty,$$

for $f : \mathbb{Q} \to \mathbb{R}$ and the sets S_n correspondingly specified.

We call a function $f : \mathbb{Q} \to \mathbb{R}$ additive, if $f(1) = 0$ and for any $r \in \mathbb{Q}$,

$$r = \prod_p p^{\beta_p}, \quad f(r) = \sum_p f(p^{\beta_p}),$$

here p stands for primes and β_p are integers.

The value distribution problems for additive functions $f : \mathbb{Q} \to \mathbb{R}$ are reduced to that ones for the correspondingly defined sequence of additive functions $f_n : \mathbb{N} \to \mathbb{R}$, here $\mathbb{N}$ is the set of natural numbers.

Estimating a Gradual Parameter Change in an AR(1)-Process

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Abstract

In this talk we present some work in progress concerning the estimation of a *change-point* at which the parameter of a (non-stationary) AR(1)-process possibly changes in a gradual way. More precisely, we observe a time series $X_1, \ldots, X_n$ possessing the structure

$$X_t = (\beta_0 + \beta_1 g(t, t_0))X_{t-1} + e_t \quad (t = 1, 2, \ldots), \quad \text{with} \quad X_0 = e_0,$$

where $\{e_t\}_{t=0,1,\ldots}$ is a sequence of centered innovations, β_0, β_1 are unknown parameters satisfying $|\beta_0| < 1$, $\beta_1 \to 0$, $\beta_1\sqrt{n} \to \infty$ $(n \to \infty)$, and $g(\cdot, t_0)$ is a (known) real function such that $g(t, t_0) = 0$ $(t \leq t_0)$ and $g(t, t_0) \neq 0$ $(t > t_0)$. That is, we assume that the parameter β_0 of the AR(1)-process changes *gradually* at an unknown time-point $t_0 = \lfloor n\tau_0 \rfloor$, with $0 < \tau_0 < 1$.

We suggest to make use of the least squares estimator $\widehat{t_0}$ for t_0, which is obtained by minimizing the sum of squares

$$S(b_0, b_1, t_*) = \sum_{t=1}^{n} [X_t - (b_0 + b_1 g(t, t_*))X_{t-1}]^2$$

with respect to $b_0, b_1 \in \mathbb{R}$, $t_* = 0, 1, \ldots, \lfloor n(1-\delta) \rfloor$, $\delta > 0$ arbitrarily small.

As a first result it can be shown that, under certain regularity and moment assumptions, $\widehat{t_0}$ is a consistent estimator for t_0, i.e., $\widehat{t_0}/n \xrightarrow{P} \tau_0$ $(n \to \infty)$. A rough convergence rate statement can also be given and some possible further investigations will be discussed, including the weak limiting behaviour of the (suitably normalized) estimator. Additionally, the results of a small simulation study will be presented to demonstrate the finite sample behaviour of the suggested estimator.

This talk is based on joint work (in progress) with Marie Hušková and Zuzana Prášková (Charles University Prague, Czech Republic).

Reference

[1] M. Hušková, Z. Prášková, J.G. Steinebach (2018+). Estimating a gradual parameter change in an AR(1)-process. Preprint, Charles University Prague and University of Cologne (in preparation)

Mean values of products of multiplicative functions

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Abstract

The global behaviour of multiplicative functions has been extensively studied and is now well understood for a large class of multiplicative functions. In particular, G. Halász completely determined the asymptotic behaviour of the means

$$\frac{1}{x}\sum_{n\le x} g(n)$$

for multiplicative functions $|g| \le 1$, and gave necessary and sufficient conditions for the existence of the mean value

$$\lim_{x\to\infty}\frac{1}{x}\sum_{n\le x} g(n).$$

In contrast to this, much less is known about the *local* behaviour of multiplicative functions, i.e. the behaviour on short sequences of consecutive integers $n, n+1, \ldots, n+s$, with fixed or slowly increasing s.

We can expect that the values $g(n), \ldots, g(n+s)$ are, in an appropriate sense, mutually independent for a "typical" multiplicative functions g. However, proving concrete results in this direction is a difficult problem, even in the simplest case $s = 1$.

A natural approach here would be to consider the averages

$$\frac{1}{x}\sum_{n\le x} g(n)g(n+1) \tag{5}$$

and to try to obtain analogues of Halász' results. Unfortunately, Halász' analytic method cannot be used to deal with (5), since the corresponding Dirichlet series do not have an Euler product representation.

During the last decades the tangible progress is done. And now, the limit behaviour of the expression (5) is intensively investigated.

We will discuss about that and focus on more recent results in this direction.

Moment of additive functions defined on random permutations

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Abstract

Since it's introduction in 1956, the Túran-Kubilius inequality

$$\sum_{n\leq x}|f(n)-A(x)|^2 \ll xB(x)^2$$

for an additive function $f:\mathbb{N}\to\mathbb{C}$, where

$$A(x):=\sum_{p^k\leq x}\frac{f(p^k)}{p^k},\ B(x)^2:=\sum_{p^k\leq x}\frac{|f(p^k)|^2}{p^k},$$

and p are prime numbers, have been extensively studied, many generalizations and extensions were made. Notably, P.D.T.A.Elliott in 1980 generalized the inequality for $\sum_{n\leq x}|f(n)-A(x)|^{\alpha}$, $\alpha\geq 0$ and K.-H.Indlekofer in 1989 generalized further for $\sum_{n\leq x}\phi\big(|f(n)-A(x)|\big)$, where $\phi:\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is monotonically increasing function, $\lim_{x\to\infty}\phi(x)=\infty$ and $\phi(x+y)\leq \frac{1}{2}c\big(\phi(x)+\phi(y)\big)$ for some $c\geq 1$ and all $x,y\geq 0$.

One can define additive function on combinatorial structures, such as permutations by using structure-of-cycle vectors and setting $H(\bar{s}+\bar{t})=H(\bar{s})+H(\bar{t})$ if $\bar{s}\perp\bar{t}$.

We will discuss analogous of the Túran-Kubilius inequality for additive functions defined on combinatorial structures and introduce some recent results.