Program of the workshop

Westfälische Stochastiktage

PADERBORN, SEPTEMBER 19-20, 2024

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Westfälische Stochastiktage - Timetable

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Dormancy in population genetics and interacting particle systems

Noemi Kurt[1]

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Abstract

Dormancy is a concept that appears in a multitude of biological systems, and refers to the reversible transition of an organism or cell into an inactive state for short or long periods of time. In recent years, mathematical modelling of dormancy has received some attention, in particular in population genetics, but also in related fields like interacting particle systems.
In this talk, we will introduce the concept of dormancy, and its various occurrences, and present some stochastic models from population genetics that include dormant forms. We will define the so-called seed bank coalescent as the genealogy of an evolving population with dormancy. We will review some of its specific properties, for example an extended time to the most recent common ancestor, and discuss different time scales. From a mathematical perspective, duality of Markov processes will play a central role. Further, we will present some examples of dormancy in interacting particle systems, in particular the contact process with dormancy.
This talk is based on joint work with Jochen Blath, Adrián González Casanova, Michel Reitmeier, and Maite Wilke Berenguer.

Alpha-stable branching and beta-frequency processes, beyond the IID assumption

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Abstract

Birkner et al. obtained necessary and sufficient conditions for the frequency process between two independent and identically distributed continuous-state branching processes time-changed by a functional of the total mass process to be a Markov process. Foucart et al. extended this result to continuous-state branching processes with immigration. We generalize these results by dropping the *independent and identically distributed* assumption. Our result clarifies under which conditions a multi-type Λ-coalescent can be constructed from a multi-type branching process by a time change using the total mass. Finally, we address a problem formulated by Griffiths, by clarifying the relation between 2-type α-stable continuous-state branching processes and 2-type β-Fleming–Viot processes with mutation and selection.

A new random polymer model drawn off the Edwards' model

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Abstract

The Edwards' model is a mathematical description for the the emergence of polymer chains, it considers a Brownian motion X to be penalized (in a suitable sense) for self intersections. In the classical one-dimensional case, this is established via the Brownian local time $L(t, x)$, which is the linked space-time process covering (in some sense) how much time X spend in $x \in \mathbb{R}$ up to $t > 0$. For fixed $T > 0$, the investigation of the space process $(L(T, x))_{x\in\mathbb{R}}$ comes with some difficulties, what leaves many questions about the model unanswered. We attempt a new approach by randomizing the time T, hence we are able to exploit the common Ray-Knight theorems, which link the Brownian local time to the well understood Bessel processes.

Random walk conditioned to stay positive

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Abstract

Consider a one-dimensional random walk S_n with i.i.d. increments with zero mean and finite variance. We study the asymptotic expansion for the tail distribution $\mathbb{P}(\tau_x > n)$ of the first passage times $\tau_x := \inf\{n \geq 1 : x + S_n \leq 0\}$ under minimal moment conditions. We also derive an asymptotic expansion for local probabilities $\mathbb{P}(S_n = x, \tau_0 > n)$ in two different regimes: for $x = o(\sqrt{n})$ and for $x \sim \sqrt{n}$.

The talk is based on joint work with V. Wachtel and D. Denisov.

Limit theorems on block p-balls

Daniel Rosen, Tristan Schiller[1], Christoph Thäle

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Abstract

We introduce the concept of block p-balls and discuss their geometry. They are a generalisation of both ℓ_p-balls and the unit balls of the mixed-norm sequence spaces. Further we discuss a family of mixture distributions on them and if time permits we present a limit theorem of your choice. For this we will use a new and cool reduction technique.

Angles of orthocentric simplices

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Abstract

A simplex is called orthocentric if all its altitudes intersect in a single point. This point is called the orthocenter. The orthocentric simplices whose orthocenter lies in the interior of the simplex can be characterized, up to an isometry, as the simplices

$$\mathrm{Conv}\left(\frac{e_0}{\tau_0},\dots,\frac{e_d}{\tau_d}\right),$$

where $d \geq 2, \tau_0,\dots,\tau_d > 0$ and $e_0,\dots,e_d$ denotes the standard orthonormal basis of $\mathbb{R}^{d+1}$.

We derive formulas for the internal and external angles of such simplices. We demonstrate how these results can be used to find the expected number of faces of the random polytope

$$\mathrm{Conv}\left(\frac{g_1}{\tau_1},\dots,\frac{g_n}{\tau_n}\right),$$

where $g_1,\dots,g_n$ are independent standard Gaussian distributed random vectors.

Fully-connected continuum percolation on $\mathbb{R}^d$

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Abstract

We are investigating the fully-connected continuum percolation on $\mathbb{R}^d$.
Visually, in the standard continuum percolation, we take a Poisson point process on $\mathbb{R}^d$, draw a circle of radius 1 around each point, and observe the resulting clusters.
Heuristically, the fully-connectivity arises from conditioning this process to have only one cluster. Since this is not possible, we consider this process in d-dimensional cubes under some boundary conditions and pass to the limit.
We have developed a two-parameter model that allows us to describe this process.

Non-Atomicity of the extremal decomposition of Gibbs states with finite localization sets for clock models on Cayley trees

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Abstract

We consider $\mathbb{Z}_q$-valued clock models on a regular tree. Recently it has been proven that, at strong enough coupling, families of homogeneous Markov chain Gibbs states μ_A coexist whose single-site marginals concentrate on $A \subset \mathbb{Z}_q$, and which are not convex combinations of each other [Abbondandolo, Henning, Külske, Majer, 2024]. The key point of our work is to aim at a description of the extremal decomposition of μ_A into all extremal Gibbs measures, which may be spatially inhomogeneous. We show that in a low temperature regime the decomposition is supported on uncountably many inhomogeneous extremal states, that we call *glassy states*.

Fractional Sobolev paths on Wasserstein spaces with a compatibility property

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Abstract

We study paths of low regularity on Wasserstein spaces. We first show that any deterministic curve on a p-Wasserstein space with fractional Sobolev regularity $W^{\alpha,p}$ and a compatibility property can be lifted to a probability measure on continuous paths concentrated on $W^{\alpha,p}$ paths, where $1/p < \alpha < 1$. The $W^{\alpha,p}$-energy of this lift coincides with $W^{\alpha,p}$-(semi)norm of the Wasserstein curve, and this norm can be realized by lifts on continuous paths only under the strong assumption of compatibility. As a corollary, we obtain a lift for γ-Hölder compatible curves on p-Wasserstein spaces, where $1/p < \gamma \leq 1$. The results are then extended to the stochastic setting by constructing a random lift from a measure-valued process satisfying analogous conditions. This provides a stochastic superposition principle based on optimal transport, generalizing the previous deterministic results by Lott–Villani for Wasserstein geodesics and by Lisini for absolutely continuous curves. Its connection to Kolmogorov–Chentsov continuity theorem is also discussed. Finally, we demonstrate via a counterexample that if the restrictive assumption of compatibility is removed, a Wasserstein curve with finite $W^{\alpha,p}$-norm can have no lift on continuous paths with finite $W^{\alpha,p}$-energy.

Coexisting of branching populations

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Abstract

Consider two one-dimensional branching populations (Z_n^1, Z_n^2) in a joint random environment. Quenched distributions of Z_n^1 and Z_n^2 are assumed independent. Thus, the dependence between populations is caused by the environment only. We are interested in the asymptotic behaviour of coexisting probability $\mathbb{P}(Z_n^2 > 0, Z_n^2 > 0)$. We are going to show that this problem is deeply connected to a two-dimensional random walk $\hat{S}_n$ conditioned to stay in a cone. $\hat{S}_n$ is the Doob h-transform of a random walk S_n having i.i.d. increments with zero mean and finite variance and killed at leaving the cone. For the process $\hat{S}_n$ we estimate the probability of coming close to the boundary of the cone. This will give us upper and lower bounds for the coexistence probability.

Gibbs point processes with non-summable pairwise interaction

David Dereudre[1]

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Abstract

In this talk, we discuss the question of Gibbs point processes in $\mathbb{R}^d$ with pairwise interactions that are not integrable at infinity. A standard example is the Riesz potential of the form $\varphi(x) = \frac{1}{|x|^s}$ where $s < d$ This setting has a long history, notably because the case $s = d-2$ corresponds to the classical Coulomb potential, which arises from electrostatic theory. We will first address the existence of the process in the infinite volume regime when a neutralizing background is introduced (this model is known as Jellium in theoretical physics). Subsequently, we will discuss the rigidity of such point processes, specifically hyperuniformity and number rigidity. We will provide a state-of-the-art review and present numerous conjectures and open problems.

Statistical parameter estimation for stochastic fiber lay-down models

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Abstract

To efficiently simulate fiber lay-down processes in industrial production processes of nonwoven textiles, models based on stochastic differential equations (SDEs) have been developed at the Fraunhofer ITWM in Kaiserslautern. Statistical parameter estimation based on the available data is a key step in practical application of these models. In this context, the two main challenges are a) the multiscale character of the estimation problem due to an incompatibility of the data with the SDE model at small scales and b) the presence of hidden variables resulting from the reconstruction of angular data. In view of these challenges, we are currently examining various parameter estimation methods with respect to their applicability and adaptability to the specific problem structure.