Program of the workshop

Westfälische Stochastiktage

PADERBORN, AUGUST 18-19, 2022

PADERBORN UNIVERSITY
The University for the Information Society

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A variational condition for uniqueness of g-measures

Diana Conache (TU München)

Abstract

g-measures are known in the literature under many names: chains of infinite order, chains with complete connections, uniform martingales or Doeblin measures. Intuitively, they can be regarded as a generalization of Markov chains to processes with infinite memory. In such a setting, the uniqueness of the stationary measure becomes a non-trivial problem. In this talk, I will review the previously known uniqueness criteria for g-measures and present a new result obtained in collaboration with N. Berger, A. Johansson and A. Öberg. I will also give a conjecture regarding the threshold between the uniqueness and non-uniqueness regimes.

Group-invariant compactifications of spaces of probability measures

Konstantin Recke [1]
Joint work with Chiranjib Mukherjee[2]

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Abstract

The space of probability measures on $\mathbb{R}^d$ is not compact in the usual weak and vague topologies. Compactifying it in the vague topology results in the space of sub-probability measures on $\mathbb{R}^d$, essentially by ignoring mass which escaped in one way or another to infinity. When studying problems with underlying *translation invariance*, the drawback of this compactification is that it does not account for this fact. For such problems it is natural to consider the space of equivalence classes $\widetilde{\mathcal{M}}_1(\mathbb{R}^d)$ under the translation action of $\mathbb{R}^d$. Mukherjee and Varadhan (2016) provided a compactification of $\widetilde{\mathcal{M}}_1(\mathbb{R}^d)$ and proved a large deviation principle for Brownian occupation measures in this space. It then turns out that the compactification takes a very natural and intuitive form, which lends itself to study other problems with underlying translation invariance. Moreover, the approach can be extended to provide compactifications of other spaces of equivalence classes of probability measures with respect to group actions. In this talk, we will introduce the main ideas behind such compactifications and some key properties. We will focus on one particular extension to actions of countable discrete groups on their corresponding spaces of probability measures by translations. Finally, we will discuss an application of this compactification to geometric group theory and the theory of operator algebras associated to groups.

Random walks and their connection to bounded harmonic functions

Luzie Kupffer[1]

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Abstract

The Poisson boundary of a random walk on a group Γ with step distribution P, uniquely (up to isometry) identifies the space of bounded harmonic functions. It is known, how the existence of a Poisson boundary depends on (Γ, P) and that the long term behaviour of the random walk determines if the boundary is trivial. Around 2000, V. Kaimanovich developed a way to check whether a given boundary is a Poisson boundary, which again depends on the long term behaviour of the random walk. So, in hope of expanding upon those results, we are currently studying the long term behaviour of random walks on groups of exponential growth more closely, using tools from large deviations theory.

Gaussian Multiplicative Chaos in the Wiener Space

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Abstract

This talk will be about Gaussian multiplicative chaos in the Wiener space. We will consider a Brownian motion moving through a space-time white noise. The goal is to show the existence of the GMC. (joined work with Rodrigo Bazaes, WWU Muenster)

Kolmogorov bounds for decomposable random variables and subgraph counting by the Stein–Tikhomirov method

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Abstract

We derive normal approximation bounds in the Kolmogorov distance for random variables possessing decompositions of Barbour, Karoński, and Ruciński (1989). We highlight the example of standardized subgraph counts in the Erdős–Rényi random graph. We prove a bound by generalizing the argumentation of Röllin (2021), who used the Stein–Tikhomirov method to prove a bound in the special case of standardized triangle counts. Our bounds match the best available Wasserstein bounds.

References:

Barbour, A. D., Karoński, M., and Ruciński, A. (1989). A central limit theorem for decomposable random variables with applications to random graphs. *J. Combin. Theory Ser. B* **47** 125–145.

Röllin, A. (2021). Kolmogorov bounds for the normal approximation of the number of triangles in the Erdős–Rényi random graph. *Probab. Engrg. Inform. Sci.* 1–27.

Moderate deviations for functionals over infinitely many Rademacher random variables

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Abstract

In this talk, moderate deviations for normal approximation of functionals over infinitely many Rademacher random variables are derived. They are based on a recently established bound for the Kolmogorov distance between a general Rademacher functional and a Gaussian random variable, continued by an intensive study of the behavior of operators from the Malliavin-Stein method along with the moment generating function of the mentioned functional. As applications, infinite weighted 2-runs and subgraph counting in the Erdös-Rényi random graph are discussed.

The Size of a shadow of a symplectic ball.

Tristan Schiller[1]

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Abstract

We will introduce a result by Alberto Abbondandolo concerning the size of a k-dimensional projection of a ball under a symplectomorphism. For this, we will present Gromov' Non-squeezing theorem, one of the fundamental theorems in symplectic geometry. Alberto's result holds in specific settings, but in general one can formulate counterexamples. Thus we will raise the question of whether this result holds for "random symplectomorphisms".

Probabilistic Limit Theorems For The Coefficients of a Class of Root-Unitary Polynomials

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Abstract

Sequences of discrete random variables are studied whose probability generating functions have representation $P_N(z)/P_N(1)$, where

$$P_N(z) = \prod_{1\leq j\leq N} \frac{1-z^{b_j}}{1-z^{a_j}},$$

for $a_j, b_j \in \mathbb{N}$ and $b_j \geq a_j$, is a polynomial whose roots are located on the unit circle in the complex plane. Since there exists a representation for the cumulants of order at least two of those random variables, bounds on these cumulants are established. This is leading to Berry-Esseen bounds, giving a rate of convergence to normal distribution in central limit theorems. Moreover, moderate deviation results, concentration inequalities and mod-Gaussian convergence is discovered.

A variety of examples naturally fit into this context, of which a few are discussed in detail, for example the number of inversions in a random permutation of N elements (and its generalization to inversions in Coxeter groups of type A_N, B_N and D_N) or (generalized) q-Catalan numbers.

Large Deviation Principle for Geometric and Topological Functionals and Associated Point Processes

Christian Hirsch (Aarhus University)

Abstract

We prove a large deviation principle for the point process associated to k-element connected components in $\mathbb{R}^d$ with respect to the connectivity radii $r_n \to \infty$. The random points are generated from a homogeneous Poisson point process, so that $(r_n)_{n\geq 1}$ satisfies $n^k r_n^{d(k-1)} \to \infty$ and $nr_n^d \to 0$ as $n \to \infty$ (i.e., sparse regime). The rate function for the obtained large deviation principle can be represented as relative entropy. As an application, we deduce large deviation principles for various functionals and point processes appearing in stochastic geometry and topology. As concrete examples of topological invariants, we consider persistent Betti numbers of geometric complexes and the number of Morse critical points of the min-type distance function. This talk is based on joint work with Takashi Owada.

Joint asymptotic distribution of point process and squared Frobenius norm of the sample covariance matrix

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Abstract

Recently, the analysis of high dimensional data has become increasingly more important. In this talk, a CLT for the squared Frobenius norm of the sample covariance matrix of p iid points in $\mathbb{R}^n$ with finite fourth moment is presented. It holds if the number of vectors p tends to infinity as the dimension n tends to infinity. In the case of infinite fourth moment, convergence in distribution towards a stable law is sustained, if the components of the random vectors are regularly varying. Asymptotic independence of the squared Frobenius norm and the point process of the entries of the sample covariance matrix is shown if the fourth moment exists.

Brownian motion with bounded local time

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Abstract

We consider a one-dimensional Brownian Motion conditioned to only spend bounded time at any given level. This behaviour is equivalent to the corresponding local times having constraints at every point and moment.
Results by I. Benjamini and N. Berestycki have shown that such a process is well defined as a limiting procedure and suffices a specific law of large numbers [1]. These results are obtained by linking the local times of a Brownian motion to squared Bessel processes via the famous Ray-knight theorems. Our goal is to adapt this strategy and use known theory about fluctuations in Markov processes in order to achieve a kind of central limit theorem for the given process.

[1] Itai Benjamini and Nathanaël Berestycki. Random paths with bounded local time. *Journal of the European Mathematical Society*, pages 819–854, 2010.

Solutions to spatial kinetic-type equations

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Abstract

In our work we consider a spatial kinetic-type evolution equation, introduced in [1]. Let $E = \{f \in C_b(\mathbb{R}^3, \mathbb{C}) : ||f||_\infty \leq 1\}$. For a time-dependent family $(f_t)_{t\geq 0} \subset E$ the equation of our interest reads

$$\begin{cases} \frac{\partial}{\partial t} f_t(\xi) + f_t(\xi) = \widehat{Q}(f_t(\cdot), f_t(\cdot))(\xi), \\ f_0(\xi) = f_0. \end{cases} \tag{1}$$

$\widehat{Q}$ is the so-called collisional gain operator. For $r \in \mathbb{R}_+$, $o \in \mathbb{O}(3)$ and $e_3 = (0,0,1)^T$ it is defined as

$$\widehat{Q}(f_t(\cdot), f_t(\cdot))(roe_3) = \mathbb{E}\left[f_t(roR_1O_1e_3)f_t(roR_2O_2e_3)\right],$$

where $R_1, R_2 \in \mathbb{R}_+$ are positive random variables and $O_1, O_2 \in \mathbb{O}(3)$ are random orthogonal 3×3 matrices. Given the initial condition $f_0 \in E$, we construct a unique solution to (1) with the help of a probabilistic model based on the Yule process, which was introduced in [2] for a similar problem in one-dimensional case. Further, we construct a stationary solution to (1).

References

[1] Bassetti, F., Ladelli, L. and Matthes, D. (2015). Infinite energy solutions to inelastic homogeneous Boltzmann equations. *Electronic Journal of Probability*, **20**, pp. 1–34, `https://doi.org/10.1214/EJP.v20-3531`.

[2] Bogus, K., Buraczewski, D. and Marynych, A. (2020). Self-similar solutions of kinetic-type equations: the boundary case. *Stochastic Processes and their Applications*, **130(2)**, pp. 677–693, ISSN 0304-4149, `https://doi.org/10.1016/j.spa.2019.03.005`.

ASYMPTOTIC EXPANSIONS FOR FIRST-PASSAGE TIMES OF A RANDOM WALK

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Abstract

For a random walk in $\mathbb{R}$ with i.i.d. increments define the first time of reaching negative value if it starts at point $x \geq 0$: $\tau_x = \inf\{n : S_n + x \leq 0\}$. The question is what we can say about the asymptotic expansions of the probability $\mathbb{P}(\tau_x \geq n)$. Previously it was mostly known the main asymptotic, but we will show that there is asymptotic decomposition of any accuracy under the condition of high moment existence.

Fully-connected continuum percolation on $\mathbb{R}^d$

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Abstract

We consider the continuum percolation on $\mathbb{R}^d$. That means we have a poisson point process on $\mathbb{R}^d$ with a ball of radius 1 around each point. If two balls are intersecting we consider these points connected. By this we obtain clusters of points. In this talk we study this process with the constraint to only have one cluster.

Results by D. Dereudre have shown that in the discrete case of a fully-connected bond percolation on $\mathbb{Z}^d$ we have a threshold such that any infinitive volume model has necessarily no open edges in the subcritical regime and is non-trivial in the supercritical regime. The question is whether there the exists a similar threshold in case of the continuum percolation.

The classical and the soft-killing inverse first-passage time problem for Brownian motion

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Abstract

The first-passage time of a process is the first time the process crosses a given time-dependent boundary. The classical first-passage time problem asks for the distribution of this stopping time. In the inverse problem the distribution is given and the task is to find a time-dependent boundary such that its first-passage time has the given distribution. The classical questions arising from this problem are the analysis of existence and uniqueness of boundary solutions and the properties of such solutions. In this talk we will be concerned with this classical problem for Brownian motion and its soft-killing variant and discuss results obtained by a new stochastic order approach. This is joint work with Prof. Dr. Martin Kolb.

Limit theory of sparse random geometric graphs in high dimensions

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Abstract

We study topological and geometric functionals of high-dimensional l_∞-random geometric graph in a sparse regime, where the expected number of neighbors decays exponentially in the dimension. More precisely, we establish moment asymptotics, central limit theorems and Poisson approximation theorems for certain functionals that are additive under disjoint unions of graphs. For instance, this includes simplex counts and Betti numbers of the Rips complex, as well as general subgraph counts of the random geometric graph. We also present multi-additive extensions that cover the case of persistent Betti numbers of the Rips complex.

Time-evolved Gibbs measures on trees

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Abstract

Gibbs-non-Gibbs transitions for interacting particle systems have been analyzed on various types of graphs, for example for the low-temperature Ising model on the two or more dimensional lattice under stochastic independent spin-flip dynamics.

We study these types of transitions for homogeneous Markovian models under stochastic independent spin-flip dynamics on trees. It is well known that, in the non time-evolved case, extremal Gibbs measures of these models on regular trees correspond to so-called tree-indexed Markov chains which are characterized by fixed point solutions of a dynamical system. This, in contrast to non-tree models, directly leads to an explicit representation of the finite-volume probabilities of these extremal Gibbs measures.

Under certain circumstances the finite-volume conditional expectations for time-evolved versions of these models can then be analyzed by investigating perturbed versions of these dynamical systems. This allows us to search for sets of discontinuity for the infinite-volume conditional expectations that have positive measure and whose existence imply non-Gibbsianness of the model.

In my talk I would like to present the general idea of this approach in more detail and discuss some possible generalizations.

Central limit theorems for generalized descents and generalized inversions in finite root systems

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Abstract

Motivated by works of Bona and Pike on d-descents, we introduce generalizations of inversions and descents first for permutations and then for any Weyl group admitting a finite crystallographic root system. We use a dependency graph method to conclude a central limit theorem for these statistics. In the proof of this theorem for generalized descents, we prove an even stronger statement about antichains of root posets.
This is joint work with Christian Stump.

A lower bound on the displacement of particles in 2D Gibbsian particle systems

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Abstract

While 2D Gibbsian particle systems might exhibit orientational order resulting in a lattice-like structure, these particle systems do not show positional order if the interaction between particles satisfies weak assumptions which guarantee the absence of positional order. In this talk we discuss to which extent particles within a box of size $2n \times 2n$ may fluctuate from their ideal lattice position. We consider particle systems with fairly arbitrary particle spins and pair interaction, e.g. Widom-Rowlinson or Lennard-Jones-type interaction. This work extends results from the second named author who achieved similar results for the hard disk model.

Moreover, we will have a look at extensions of these results for certain many-body interactions. However this is still work in progress.