

Classical Diophantine approximation quantifies the denseness of the set of rational vectors in their ambient Euclidean space. A far-reaching extension of the classical theory is to pose the challenge of quantifying the denseness of rational points (and rational points with constrained denominators) in general homogeneous algebraic varieties. This was raised as an open problem by Serge Lang already half a century ago, but progress towards it was achieved only in a limited number of special cases. A systematic approach to this problem for homogeneous varieties associated with semi-simple groups has been developed in recent years, in joint work with A. Ghosh and A. Gorodnik. The methods employ dynamical arguments and effective ergodic theory, based on unitary representation theory of semi-simple groups. In the case of reductive homogeneous spaces with semi-simple stability group, this approach leads to the derivation of uniform and almost sure Diophantine exponents, as well as analogs of Khinchin's and Schmidt's theorems, with some of the results being best possible. We will explain some of the main results and some of the ingredients in their proof, focusing on some easily accessible examples.