Previous conferences in this series

- 17th British-French-German Conference on Optimization, 2015, London, UK.
- 16th French-German-Polish Conference on Optimization, 2013, Kraków, Poland.
- 15th Austrian-French-German Conference on Optimization, 2011, Toulouse, France.
- 14th Belgian-French-German Conference on Optimization, 2009, Leuven, Belgium.
- 13th Czech-French-German Conference on Optimization, 2007, Heidelberg, Germany.
- 12th French-German-Spanish Conference on Optimization, 2004, Avignon, France.
- 11th French-German-Polish Conference on Optimization, 2002, Cottbus, Germany.
- 10th French-German-Italian Conference on Optimization, 2000, Montpellier, France.
- 9th Belgian-French-German Conference on Optimization, 1998, Namur, Belgium.
- 8th French-German Conference on Optimization, 1996, Trier, Germany.
- 7th French-German Conference on Optimization, 1994, Dijon, France.
- 6th French-German Conference on Optimization, 1991, Lambrecht, Germany.
- 5th French-German Conference on Optimization, 1988, Varetz, France.
- 4th French-German Conference on Optimization, 1986, Irsee, W. Germany.
- 3rd French-German Conference on Optimization, 1984, Luminy, France.
- Optimization: Theory and Algorithms, 1981, Confolant, France.
- Optimization and Optimal Control, 1980, Oberwolfach, W. Germany.

Dear conference participants,

welcome to the 18th French-German-Italian Conference on Optimization taking place at Paderborn University. We hope that the conference will provide a platform for interchanging ideas, research results and experiences for an international community, actively interested in optimization.

The optimization conference is one of a series with a long and distinguished history which started in Oberwolfach in 1980. Since 1998 the conference has been organized with the participation of a third European country, and this year the conference takes place in Germany for the 7th time. The aim of the conference is to provide a forum for theoreticians and practitioners from academia and industry to exchange knowledge, ideas and results in a broad range of topics relevant to the theory and practice of optimization methods, models, software and applications in engineering, finance and management.

We would like to thank our sponsor DFG Bonn.

We hope you enjoy the conference and wish you a pleasant stay in Paderborn.

With our compliments,

Andrea Walther on behalf of the programme committee.

Contents

1 General Information **1**

Programme 3

2 Monday Programme **3**

3 Tuesday Programme **5**

4 Wednesday Programme **8**

5 Thursday Programme **11**

Abstracts 14

6 Invited Speakers **14**

7 Minisymposia **19**

MS 1 Linear algebra issues in optimization methods . . . 19
MS 2 Numerical optimal control: Methods I . . . 23
MS 3 Numerical optimal control: Methods II . . . 26
MS 4 Numerical optimal control with PDE constraints . . . 28
MS 5 Numerical optimal control: Applications . . . 31
MS 6 Methods for continuous optimization and applications . . . 34
MS 7 Optimization in finance . . . 35
MS 8 Variational problems and their interdependences . . . 37
MS 8a Part I . . . 37
MS 8b Part II . . . 39
MS 9 Model order reduction in PDE-constrained optimization . . . 42
MS 10 New hierarchies of SDP relaxations for polynomial systems . . . 44
MS 11 First order methods and applications . . . 47
MS 12 Optimization methods for inverse problems in imaging . . . 49
MS 13 Geometric control and applications . . . 54
MS 14 Non-smooth optimization and computational 3D image . . . 56

8 Contributed Talks **60**

CT 1 Vector-value optimization and related issues . . . 60
CT 2 PDE-based optimization . . . 64
CT 3 Optimization algorithms I . . . 68
CT 4 Nonsmooth Optimization . . . 71
CT 5 Robust optimization/Optimization under uncertainty . . . 74
CT 6 Optimization methods II . . . 79

Author Index **83**

1. General Information

Venue

The conference is taking place on campus of the Paderborn University, located south of the city center. All the talks will be held in Building L, which is located at the crossing of the streets *Südring* and *Pohlweg* towards the north east, opposite from the shopping complex *Südring Center*.

All the invited lectures shall be held in the lecture hall L1, which is accessible from both the ground floor and the first floor. The minisymposia and contributed talks shall take place in lecture hall L1, and in rooms L1.201 and L1.202, located on the first floor.

The location of Building L and the Mensa on the campus map, locations of some restaurants in the city and the bus network plan can be seen in the maps on pages 85–87.

Presentation Instructions

Plenary talks last one hour including set up and questions. All presentations that are part of a minisymposium last 30 minutes including setup and questions. All contributed talks last 20 minutes including set up and questions. Please bring a copy of your presentation on a USB stick and in PDF format in order to avoid compatibility problems. In case you wish to use your own laptop for your presentation, please make sure you also bring the appropriate adapter for connecting with a VGA cable.

Internet Access

If you use the wireless network ***eduroam*** at your home institution you can use this network at the Paderborn University without any modification. Otherwise please contact the registration-desk to receive a personalized login and password for the wireless network ***webauth*** along with instructions to connect. Please be aware that we are required by law to retain the names of all the network users for at least 2 months.

Social Events & Additional Information

Registration: Monday, 25 September, 11:30am - 12:30pm

Registration will take place in the foyer of lecture hall L1 (Building L, see Campus Map).

Conference dinner: Tuesday, 26 September, 8pm - 10pm

The conference dinner takes place in the Mensa Forum on the ground floor of Building ME (see Campus Map).

Reception at townhall: Wednesday, 27 September, 6pm - 8:30pm

Lunch: Lunch is available in the Mensa Academica on the first floor of Building ME (see Campus Map). All participants will get lunch tickets for Tuesday, Wednesday and Thursday at registration.

Coffee breaks: During the coffee breaks light refreshments will be served in the foyer of lecture hall L1.

Toilets: Men's and women's toilets are located just next to the foyer.

Scientific Committee

- Andrei A. Agrachev, Trieste, Italy
- Stefania Bellavia, Firenze, Italy
- Dorin Bucur, Chambéry, France
- Francisco Facchinei, Roma, Italy
- Maurizio Falcone, Roma, Italy
- Matteo Fischetti, Padova, Italy
- Rida Laraki, Paris, France
- Alexander Martin, Erlangen-Nürnberg, Germany
- Helmut Maurer, Münster, Germany
- Dominikus Noll, Toulouse, France
- Sabine Pickenhain, Cottbus, Germany
- Filippo Santambrogio, Orsay, France
- Emmanuel Trélat, Paris, France
- Michael Ulbrich, München, Germany
- Andrea Walther, Paderborn, Germany

Organizing Commitee

- Andrea Walther
- Karin Senske
- Olga Ebel
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- Kshitij Kulshreshtha
- Veronika Schulze

Sponsors

- Deutsche Forschungsgemeinschaft (DFG), Bonn

DFG Deutsche Forschungsgemeinschaft

Programme

2. Monday Programme

11:30-12:30. **Registration.** Foyer, Building L
12:30-13:00. **Opening.** Lecture hall L1, Building L

Invited Lecture: Jacek Gondzio, University of Edinburgh
Cutting Planes and Column Generation with the Primal-Dual Interior Point Method
Time: 13:00-14:00; **Room:** L1, Building L
Chair: Jean-Baptiste Hiriart-Urruty, Université Paul Sabatier de Toulouse

Invited Lecture: Marco Sciandrone, Universita di Firenze
New Developments in Multiobjective Optimization
Time: 14:00-15:00; **Room:** L1, Building L
Chair: Giancarlo Bigi, Università di Pisa

Coffee Break
Time: 15:00-15:30

Minisymposium 4:
Numerical optimal control with PDE constraints
Time: 15:30 - 17:30
Room: L1, Building L
Chair and Organiser: Kathrin Flaßkamp, Universität Bremen, Kurt Chudej, Universität Bayreuth, and Matthias Gerdts, Universität der Bundeswehr, München
Speakers:
15:30 - 16:00 Falk Hante
(Friedrich-Alexander-Universität Erlangen-Nürnberg)
Gradient descent methods for optimization with mixed-integer and PDE constraints

Minisymposium 11:
First order methods and applications
Time: 15:30 - 17:30
Room: L1.202, Building L
Chair and Organiser: Gerardo Toraldo (Università degli Studi di Napoli Federico II)
Speakers:
15:30 - 16:00 Vanna Lisa Coli
(Universita di Modena e Reggio Emilia):
First-order scaled methods for image reconstruction in X-rays Computed Tomography
16:00 - 16:30 Marco Viola
(Universita di Roma la Sapienza):
P2GP: a proportioning-based two-phase gradient algorithm for QP problems with bounds on the variables and a linear constraint

Minisymposium 13:
Geometric control and applications
Time: 15:30 - 17:30
Room: L1.201, Building L
Chair and Organiser: Jean-Baptiste Caillau, Université Bourgogne Franche-Comté
Speakers:
15:30 - 16:00 Laura Poggiolini
(Università degli Studi di Firenze)
Bang–bang–singular extremals in Mayer problems
16:00 - 16:30 Michael Orieux
(Université Paris-Dauphine)
Pi-Singularities in minimum time control applied to space mechanic

16:00 - 16:30 Duncan Gathungu
(Universität Würzburg)
A multigrid scheme for solving convection-diffusion-integral optimal control problems

16:30 - 17:00 Mario Annunziato
(Università degli Studi di Salerno)
A numerical solver for the Fokker-Planck optimal control of stochastic jump-diffusion processes

16:30 - 17:00 Oldrich Vlach
(Technical University of Ostrava):
QP algorithms with rate of convergence and applications

16:30 - 17:00 Jeremy Rouot
(LAAS Toulouse)
Averaging for minimum time control problems and applications

Invited Lecture: Quentin Mérigot, Université Paris-Sud
Semi-discrete approach to optimal transport and Monge-Ampère equations
Time: 17:30 - 18:30, **Room:** L1, Building L
Chair: Jean-Baptiste Caillau, Université Bourgogne Franche-Comté

3. Tuesday Programme

Invited Lecture: Alexandra Schwartz, Technische Universität Darmstadt
A nonlinear approach to sparse optimization
Time: 8:30 - 9:30; **Room:** L1, Building L
Chair: Luca Zanni, Universita di Modena e Reggio Emilia

Invited Lecture: Pierre Apkarian, ONERA Toulouse
Feedback Control Meets Non-smooth Optimization
Time: 9:30 - 10:30, **Room:** L1, Building L
Chair: Sabine Pickenhain, Brandenburgische Technische Universität Cottbus

Coffee Break and a small foto shooting for the conference article
Time: 10:30 - 11:00; **Place:** Infront of building L

Minisymposium 2:
Numerical optimal control: Methods I
Time: 11:00 - 13:00
Room: L1, Building L
Chair and Organiser: Kathrin Flaßkamp, Universität Bremen, Kurt Chudej, Universität Bayreuth, and Matthias Gerdts, Universität der Bundeswehr, München
Speakers:
11:00 - 11:30 Sven-Joachim Kimmerle
(Universität der Bundeswehr)
Optimal control of a coupled system of hyperbolic and ordinary differential equations: FDTO and FOTD approaches and structure-exploiting SQP algorithms
11:30 - 12:00 Christopher Schneider
(Friedrich-Schiller-Universität Jena)
Group sparsity in optimal control
12:00 - 12:30 Kathrin Flaßkamp
(Universität Bremen)
Structure-preserving discretization in optimal control

Minisymposium 8a:
Variational problems and their interdependences Part I
Time: 11:00 - 13:00
Room: L1.201, Building L
Organiser: Giancarlo Bigi, Università di Pisa, and Simone Sagratella, Sapienza Università di Roma
Chair: Giancarlo Bigi, Università di Pisa
Speakers:
11:00 - 11:30 Andreas Fischer
(Technische Universität Dresden)
Newton-type methods for Fritz-John systems of generalized Nash equilibrium problems
11:30 - 12:00 Daniel Steck
(Universität Würzburg)
Generalized Nash equilibria and their computation via augmented Lagrangian methods
12:00 - 12:30 Simone Sagratella
(Sapienza Universita di Roma)
A bridge between bilevel programs and Nash games

Minisymposium 12:
Optimization methods for inverse problems in imaging
Time: 11:00 - 13:00
Room: L1.202, Building L
Chair and Organiser: Elena Loli Piccolomini, University of Bologna, and Luca Zanni, University of Modena and Reggio Emilia
Speakers:
11:00 - 11:30 Jack Spencer
(University of Liverpool)
On optimization of a network of minimal paths for 3D image segmentation
11:30 - 12:00 Germana Landi
(University of Bologna)
Automatic adaptive multi-penalty regularization for linear inverse problems
12:00 - 12:30 Jean-Christophe Pesquet
(University Paris-Saclay)
A variational Bayesian approach for image restoration with Poisson-Gaussian noise

12:30 - 13:00 Kurt Chudej
(Universität Bayreuth)
Optimal vaccination strategies against Dengue fever

12:30 - 13:00 Simone Rebegoldi
(University of Modena and Reggio Emilia)
A block-coordinate variable metric line-search based proximal-gradient method for nonconvex optimization

Lunch
Time: 13:00-14:00; **Place:** Mensa Academica

Contributed talks 1:
Vector-value optimization and related issues
Time: 14:00-16:00
Room: L1.202, Building L:
Chair: Nicolae Popovici, Babes-Bolyai University
Speakers:
14:00 - 14:20 Nicolae Popovici
(Babes-Bolyai University)
Local-global type properties for generalized convex vector functions
14:20 - 14:40 Enrico Moretto
(University of Insubria)
Robust counterparts of the Markowitz portfolio optimization problem via utility functions
14:40 - 15:00 Sorin-Mihai Grad (Technische Universität Chemnitz)
A forward-backward method for solving vector optimization problems
15:00 - 15:20 Christian Günther
(Martin-Luther-Universität Hallo-Wittenberg)
Jahn-Graef-Younes type algorithms for discrete vector optimization based on cone-monotone sorting functions

Contributed talks 2:
PDE-based optimization
Time: 14:00 - 16:00
Room: L1, Building L
Chair: Dörte Jando, Universität Heidelberg
Speakers:
14:00 - 14:20 Anna Walter
(Technische Universität Darmstadt)
Optimal control of elastoplastitcity problems with finite deformations and application to deep drawing processes
14:20 - 14:40 Caroline Löbhard
(Humboldt Universität zu Berlin)
Space-time discretization of a parabolic optimal control problem with state constraints
14:40 - 15:00 Andreas Potschka
(Universität Heidelberg)
Backward step control globalization for Newton-type methods
15:00 - 15:20 Mladen Banovic
(Universität Paderborn)
Automatic insertion of casing holes in TU Berlin TurboLab stator blade with differentiated Open CASCADE Technology CAD system

Contributed talks 3:
Optimization algorithms I
Time: 14:00 - 16:00
Room: L1.201, Building L
Chair: Marc Steinbach, Universität Hannover
Speakers:
14:00 - 14:20 Renke Kuhlmann
(Universität Bremen)
A primal-dual augmented Lagrangian penalty-interior-point filter line search algorithm
14:20 - 14:40 Jan Kuřátko
(Charles University)
LDLT factorization of saddle-point matrices in nonlinear optimization - reusing pivots and monitoring stability
14:40 - 15:00 Morteza Kimiaei
(Universität Wien) Comparing solvers for unconstrained and box constrained optimization
15:00 - 15:20 Jean-Baptiste Hiriart-Urruty
(Université Paul Sabatier de Toulouse)
From least squares solutions of linear inequality systems to convex least squares problems

15:20 - 15:40 Claudia Adams
(Universität Trier)
Copositive optimization in infinite dimension

15:40 - 16:00 Sebastian Peitz
(Universität Paderborn)
Multiobjective optimization with uncertainties and the application to reduced order models for PDEs

15:20 - 15:40 Hamdullah Yücel
(Middle East Technical University)
A local discontinuous Galerkin method for Dirichlet boundary control problems

15:40 - 16:00 Benjamin Jurgelucks
(Universität Paderborn)
Increasing Accuracy in Material Parameter Estimation with Algorithmic Differentiation

15:20 - 15:40 Kshitij Kulshreshtha
(Universität Paderborn)
Algorithmic differentiation for machine learning applications in Python

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Coffee Break
Time: 16:00-16:30

Invited Lecture: Karl Kunisch, Karl-Franzens-Universität Graz and RICAM
Theory and Numerical Practice for Optimization Problems Involving Functionals of l^p and L^p Type with $p \in [0, 1)$
Time: 16:30 - 17:30; **Room:** L1, Building L
Chair: Helmut Maurer, Universität Münster

Invited Lecture: Gianluigi Rozza, SISSA Trieste
Reduced Order Methods for Optimisation and Flow Control Parametric Problems in Marine Science and Engineering
Time: 17:30 - 18:30; **Room:** L1, Building L
Chair: Michael Ulbrich, Technische Universität München

Conference dinner
Time: 20:00-22:00; **Place:** Mensa Forum

4. Wednesday Programme

Invited Lecture: Daniel Kuhn, EPFL
"Dice"-sion Making under Uncertainty: When Can a Random Decision Reduce Risk?
Time: 8:00 - 9:00; **Room:** L1, Building L
Chair: Jörn Saß, Technische Universität Kaiserslautern

Invited Lecture: Michelangelo Conforti, Univerità di Padova
The Infinite Models in Integer Programming
Time: 9:00 - 10:00; **Room:** L1, Building L
Chair: Elena Loli Piccolomini, University of Bologna

Coffee Break
Time: 10:00-10:30

Minisymposium 1:
Linear algebra issues in optimization methods
Time: 10:30 - 12:30
Room: L1.202, Building L
Organiser: Martin Stoll, MPI Magdeburg, and Margherita Porcelli, Università degli Studi di Firenze
Chair: Martin Stoll, MPI Magdeburg
Speakers:
10:30 - 11:00 Daniela di Serafino
(University of Campania Luigi Vanvitelli)
Constraint-preconditioned Krylov methods for regularized saddle-point systems
11:00 - 11:30 John Pearson
(University of Kent)
Linear algebra for time-dependent PDE-constrained optimization
11:30 - 12:00 Lena Vestweber
(TU Braunschweig)
Douglas-Rachford iterations for two and three dimensional TV-, TGV- and constrained TGV denoising

Minisymposium 3:
Numerical optimal control: Methods II
Time: 10:30 - 12:30
Room: L1, Building L
Organiser: Kathrin Flaßkamp, Universität Bremen, Kurt Chudej, Universität Bayreuth, and Matthias Gerdts, Universität der Bundeswehr, München
Chair: Sven-Joachim Kimmerle, Universität der Bundeswehr München
Speakers:
10:30 - 11:00 Karl Worthmann
(Technische Universität Illmenau)
On the approximation of infinite horizon optimal control problems
11:00 - 11:30 Arthur Fleig
(Universität Bayreuth)
On model predictive control for probability density functions
11:30 - 12:00 Katharina Kolo
(Brandenburgischen Technischen Universität Cottbus–Senftenberg)
A dual based pseudospectral method for infinite horizon optimal control problems

Minisymposium 10:
New hierarchies of SDP relaxations for polynomial systems
Time: 10:30 - 12:30
Room: L1.201, Building L
Chair and Organiser: Victor Magron, CNRS Verimag Grenoble
Speakers:
10:30 - 11:00 Victor Magron
(CNRS Verimag Grenoble)
Interval enclosures of upper bounds of round off errors using semidefinite programming
11:00 - 11:30 Marcelo Forets
(Université Grenoble Alpes)
Semidefinite characterization of invariant measures for polynomial systems
11:30 - 12:00 Tillmann Weisser
(LAAS-CNRS)
Solving nearly-sparse polynomial optimization problems

12:00 - 12:30 Justin Buhendwa Nyenyezi
(University of Namur)
Preconditioning linear systems from deformable 3D medical image registration using low-rank tensor train format

12:00 - 12:30 Cedric Josz
(INRIA)
Moment/sum-of-squares hierarchy for complex polynomial optimization

Lunch
Time: 12:30-13:30; **Place:** Mensa Academica

Contributed talks 4:
Nonsmooth optimization
Time: 13:30 - 15:30
Room: L1.202, Building L
Chair: Tuomo Valkonen, University of Liverpool
Speakers:
13:30 - 13:50 Lisa Hegerhorst
(Universität Hannover)
Optimality conditions for optimization problems with nonsmooth constraints
13:50 - 14:10 Manilo Gaudioso
(Universita della Calabria)
DC programming via piecewise linear approximations
14:10 - 14:30 Thi Minh Tam Nguyen
(University of Lorraine)
DC programming and DCA for solving quadratic programs with linear complementarity constraints
14:30 - 14:50 Jose Vidal-Nunez
(Technische Universität Chemnitz)
Functions of bounded variation on nonsmooth surfaces

Contributed talks 5:
Robust optimization/Optimization under uncertainty
Time: 13:30 - 15:30
Room: L1, Building L
Chair: Andreas Potschka, Universität Heidelberg
Speakers:
13:30 - 13:50 Philip Kolvenbach
(Technische Universität Darmstadt)
Nonlinear robust optimization of PDE-constrained problems using second-order approximations
13:50 - 14:10 Patrick Schmidt
(Technische Universität Illmenau)
A smoothing approach to chance constrained optimization of parabolic PDE systems under uncertainty: A case study
14:10 - 14:30 Qaisra Fazal
(Universität Wien)
Optimization techniques for parametric differential equations
14:30 - 14:50 Moritz Schulze Darup
(Universität Paderborn)
Optimization-free robust MPC around the terminal region

Contributed talks 6:
Optimization methods II
Time: 13:30 - 15:30
Room: L1.201, Building L
Chair: Andreas Fischer, Technische Universität Dresden
Speakers:
13:30 - 13:50 Tiago Montanher
(Universität Wien)
Rigorous n unit squares packing
13:50 - 14:10 Ovidiu Bagdasar
(University of Derby)
Transportation management: methods and algorithms for solution
14:10 - 14:30 Annabella Astorino
(Istituto di Calcolo e Reti ad Alte Prestazioni)
Maximizing wireless directional sensor networks lifetime. A Lagrangian relaxation approach
14:30 - 14:50 Marc Steinbach
(Universität Hannover)
An elastic primal active set method for large QPs

14:50 - 15:10 Enrico Gorgone (University of Cagliari) Fast gradient method with dynamic smoothness parameter **15:10 - 15:30 Martin Stoll** (MPI Magdeburg) Semi-supervised learning using phase field models on graphs and hypergraphs	**14:50 - 15:10 Arthur Le Rhun** (IFP Energies nouvelles) A stochastic gradient method for Wasserstein barycenters of distributions **15:10 - 15:30 Pavel Dvurechensky** (WIAS Institut) Gradient method with inexact oracle for composite non-convex optimization	**14:50 - 15:10 Raka Jovanovic** (Hamad bin Khalifa University) A mixed integer program for the spanning distribution forest problem **15:10 - 15:30**

Coffee Break
Time: 15:30-16:00

Invited Lecture: Carola-Bibiane Schönlieb, University of Cambridge
Choose your path wisely: iterative regularisation in a Bregman distance framework
Time: 16:00-17:00; **Room:** L1, Building L
Chair: Stephan Schmidt, Universität Würzburg

Reception at townhall
Time: 18:00-20:30

5. Thursday Programme

Minisymposium 5:
Numerical optimal control: Applications
Time: 8:00 - 10:00
Room: L1, Building L
Organiser: Kathrin Flaßkamp, Universität Bremen, Kurt Chudej, Universität Bayreuth, and Matthias Gerdts, Universität der Bundeswehr, München
Chair: Sven-Joachim Kimmerle, Universität der Bundeswehr München
Speakers:
8:00 - 8:30 Helmut Maurer
(Universität Münster)
Optimal control of a global model of climate change with adaptation and mitigation
8:30 - 9:00 Kai Schäfer
(Universität Bremen)
Parameter identification for robotic systems using optimal control techniques
9:00 - 9:30 Riccardo Bonalli
(ONERA Frankreich)
The Dubins car problem with delay and applications to aeronautics motion planning problems

Minisymposium 6:
Methods for continuous optimization and applications
Time: 8:00 - 10:00
Room: L1.202, Building L
Organiser: Stefania Bellavia, Università degli Studi di Firenze, and Benedetta Morini, Università degli Studi di Firenze
Chair: Stefania Bellavia, Università degli Studi di Firenze
Speakers:
8:00 - 8:30 Christian Kanzow
(Universität Würzburg)
Modified augmented Lagrangian methods in finite and infinite dimensions
8:30 - 9:00 Natasa Krklec Jerinkic
(Novi Sad University)
Distributed Newton-like methods with variable number of working nodes
9:00 - 9:30 Margherita Porcelli
(Università degli Studi di Firenze)
Quasi-Newton methods for constrained nonlinear systems and application to gas distribution networks
9:30 - 10:00 Nico Strasdat
(Technische Universität Dresden)
Newton's method for the primal training of support vector machines

Minisymposium 14:
Non-smooth optimization and computational 3D image processing
Time: 8:00 - 10:00
Room: L1.201, Building L
Chair and Organiser: Stephan Schmidt, Universität Würzburg, and Jose Vidal Nuñez, Technische Universität Chemnitz
Speakers:
8:00 - 8:30 Tuomo Valkonen
(University of Liverpool)
Block-proximal methods with spatially adapted acceleration
8:30 - 9:00 Kristian Bredies
(Universität Graz)
Accelerated and preconditioned Douglas-Rachford algorithms for the solution of variational imaging problems
9:00 - 9:30 Marc Herrmann
(Universität Würzburg)
Total variation image reconstruction on smooth surfaces
9:30 - 10:00 Ferdia Sherry
(University of Cambridge)
Learning the sampling pattern for MRI

Coffee Break
Time: 10:00-10:30

Minisymposium 7:
Optimization in finance
Time: 10:30 - 12:30
Room: L1.202, Building L
Organiser: Marianna De Santis, Sapienza University of Rome, and Valentina De Simone, University of Campania "Luigi Vanvitelli"
Chair: Valentina De Simone, University of Campania "Luigi Vanvitelli"
Speakers:
10:30 - 11:00 Stefania Corsaro
(University of Naples Parthenope)
Bregman iteration for portfolio selection
11:00 - 11:30 Fabio Tardella
(Sapienza University of Rome)
Optimization meets diversification for portfolio selection
11:30 - 12:00 Enrico Schumann
(CPV/CAP, Switzerland)
Heuristics for portfolio selection
12:00 - 12:30 Jörn Sass
(Technische Universität Kaiserslautern)
Trading regions and structure of optimal portfolio policies under transaction costs

Minisymposium 8b:
Variational problems and their interdependences Part II
Time: 10:30 - 12:30
Room: L1.201, Building L
Organiser: Giancarlo Bigi, Università di Pisa, and Simone Sagratella, Sapienza Università di Roma
Chair: Simone Sagratella, Sapienza Università di Roma
Speakers:
10:30 - 11:00 Max Bucher
(Technische Universität Darmstadt)
Regularization for a complementarity formulation of cardinality constrained pptimization problems
11:00 - 11:30 Giancarlo Bigi
(Universita di Pisa)
Generalized Nash equilibria and semi-infinite programming

Minisymposium 9:
Model order reduction in PDE-constrained optimization
Time: 10:30 - 12:30
Room: L1, Building L
Organiser: Stefan Banholzer, Universität Konstanz, Dörte Jando, Universität Heidelberg, and Stefan Volkwein, Universität Konstanz
Chair: Dörte Jando, Universität Heidelberg
Speakers:
10:30 - 11:00 Dörte Jando
(Ruprecht-Karls-Universität Heidelberg)
Reduced order modeling for time-dependent optimal control problems with variable initial values
11:00 - 11:30 Martin Grepl
(RWTH Aachen)
Reduced basis approximation and a posteriori error bounds for 4D-var data assimilation
11:30 - 12:00 Laura Iapichino
(Eindhoven University of Technology)
Greedy controllability of reduced-order linear dynamical systems
12:00 - 12:30 Nicolas Scharmacher
(Universität Hamburg)
Adaptive trust-region POD for optimal control of the Cahn-Hilliard equation

Lunch
Time: 12:30-13:30; **Place:** Mensa Academica

Invited Lecture: Christoph Buchheim, Technische Universität Dortmund
Combinatorial Optimization under Nonlinear PDE-constraints
Time: 13:30 - 14:30; **Room:** L1, Building L
Chair: Stefania Bellavia, Università degli Studi di Firenze

Invited Lecture: Helmut Harbrecht, Universität Basel
Shape Optimization for Free Boundary Problems: Analysis and Numerics
Time: 14:30 - 15:30; **Room:** L1, Building L
Chair: Kristian Bredies, Universität Graz

Closing
Time: 15:30-16:00

End of Conference

Abstracts

6. Invited Speakers

Monday, Time: 13.00-14.00, Room: Hall L1, Building L

Cutting Planes and Column Generation with the Primal-Dual Interior Point Method

J. Gondzio[1]

[1]University of Edinburgh

Advantages of interior point methods (IPMs) applied in the context of nondifferentiable optimization arising in cutting planes/column generation applications will be discussed. Some of the many false views of the combinatorial optimization community on interior point methods applied in this context will be addressed and corrected. In particular, IPMs deliver a natural stabilization when restricted master problems are solved and guarantee fast convergence, measured with merely a few master iterations needed to localize the solution.

Several new features of the approach such as the use of primal-dual regularization and efficient IPM warm starts will be discussed. Computational experience obtained with the Primal-Dual Column Generation Method (PDCGM) software: `http://www.maths.ed.ac.uk/~gondzio/software/pdcgm.html` will be reported.

This is a joint work with Pablo Gonzalez-Brevis and Pedro Munari.

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Monday, Time: 14.00-15.30, Room: Hall L1, Building L

New developments in multiobjective optimization

M. Sciandrone[1]

[1]Universita di Firence

In this talk classes of smooth multiobjective optimization problems will be considered and steepest descent algorithms will be presented. More specifically,derivative-free algorithms for multiobjective constrained optimization, based on the implicit filtering approach, will be proposed. Furthermore, concave programming-based algorithms for multiobjective sparse optimization will be defined. The convergence analysis of the proposed algorithms will be presented, and the results of computational experiments will be shown. Finally, applications in machine learning and portfolio optimization will be analyzed and discussed.

Monday, Time: 17.30-18.30, Room: Hall L1, Building L

Semi-discrete approach to optimal transport and Monge-Ampère equations

Q. Mérigot[1]

[1]Université Paris Sud

Many problems in geometric optics or convex geometry can be recast as optimal transport problems: this includes the far-field reflector problem, Alexandrov's curvature prescription problem, etc. A popular way to solve these problems numerically is to assume that the source probability measure is absolutely continuous while the target measure is finitely supported. We refer to this setting as semi-discrete. In this talk, I will present a damped Newton's algorithm for semi-discrete optimal transport, applications of this algorithm to geometric optics, and its adaptation to Monge-Ampère equations with a non-linear right-hand side.

Tuesday, Time: 8.30-9.30, Room: Hall L1, Building L

A nonlinear approach to sparse optimization

A. Schwartz[1]

[1]TU Darmstadt

Sparse optimization problems and optimization problems with cardinality constraints have many applications such as portfolio optimization, subset selection, compressed sensing or learning. In the past solution approaches have mostly focused on convex substitutes of the respective problems, e.g. using the l1-norm to induce sparsity. However, recently nonconvex formulations have gained popularity both in learning and in the handling of sparse or cardinality constrained problems. In this talk, we give an overview over these new nonconvex reformulations and related solution approaches.

Tuesday, Time: 9.30-10.30, Room: Hall L1, Building L

Feedback control meets non-smooth optimization

P. Apkarian[1]

[1]ONERA Toulouse

Feedback control can be viewed as designing a control mechanism to achieve stability and performance for a system described by ODEs or more general classes of differential equations. A simple formulation consist in computing a controller such that the loop formed by the system and the controller is stable and meets a variety of performance requirements. Performance typically refers to damping improvement, reference tracking or disturbance and noise rejection, etc. The emergence of specialized non-smooth optimization techniques provides a novel approach to solving feedback design problems. This talk discusses different instances of the feedback design problem and shows how non-smooth optimization was used to overcome classical bottlenecks. Various real-world applications are introduced to illustrate its impacts for control engineers.

Tuesday, Time: 16:30-17:30, Room: Hall L1, Building L

Theory and numerical practice for optimization problems involving functionals of l^p type with $p \in [0,1)$

K. Kunisch[1]

[1]University of Graz

Nonsmooth nonconvex optimization problems involving the ℓ^p quasi-norm, $p \in [0,1)$ are the focus of this talk. Two schemes are presented, and analysed, and their performance in practice is discussed: A monotonically convergent scheme and a primal dual active set scheme. The latter heavily relies on a non-standard formulation of the first order optimality conditions. Numerical tests include an optimal control problem, models from fracture mechanics and microscopy image reconstruction.

Tuesday, Time: 17.30-18.30, Room: Hall L1, Building L

Reduced order methods for optimisation and flow control parametric problems in marine science and engineering

G. Rozza[1]

[1]SISSA Trieste

In this work we propose reduced order methods as a suitable approach to face parametrized optimal control problems governed by partial differential equations, with applications in environmental marine sciences and engineering. Environmental parametrized optimal control problems are usually studied for different configurations described by several physical and/or geometrical parameters representing different phenomena. Treating this kind of issue requires a demanding computational effort. Reduced basis techniques are a reliable and rapid tool to solve them, in order to save computational costs in time and memory. After a brief introduction to general parametrized linear quadratic optimal control problems, exploiting their saddle-point structure and a POD-Galerkin sampling and projection algorithm, we propose two applications: a pollutant control in the Gulf of Trieste, Italy and a solution tracking governed by quasi-geostrophic equations describing North Atlantic Ocean dynamic. The two experiments underline how reduced order methods may be a reliable and convenient tool to manage several environmental optimal control problems, differing in equations used, geographical scale and physical meaning. Time permitting some parametric shape optimisation problems in naval and nautical engineering will be shown, as well as reduction techniques in the parameter space. This is a joint work with Maria Strazzullo, Francesco Ballarin for the optimal flow control and Andrea Mola, Filippo Salmoiraghi and Marco Tezzele for shape optimisation.

Wednesday, Time: 8.00-9.00, Room: Hall L1, Building L

"Dice"-sion Making under Uncertainty: When Can a Random Decision Reduce Risk?

D. Kuhn[1]

[1]EPFL Lausanne

Stochastic programming and distributionally robust optimization seek deterministic decisions that optimize a risk measure, possibly in view of the most adverse distribution in an ambiguity set. We investigate under which circumstances such deterministic decisions are strictly outperformed by random decisions which depend on a randomization device producing uniformly distributed samples that are independent of all uncertain factors affecting the decision problem. We find that in the absence of distributional ambiguity, deterministic decisions are optimal if both the risk measure and the feasible region are convex, or alternatively if the risk measure is mixture-quasiconcave. Several classes of risk measures, such as mean (semi-)deviation and mean (semi-)moment measures, fail to be mixture-quasiconcave and can therefore give rise to problems in which the decision maker benefits from randomization. Under distributional ambiguity, on the other hand, we show that for any ambiguity averse risk measure there always exists a decision problem (with a non-convex, e.g., mixed-integer, feasible region) in which a randomized decision strictly dominates all deterministic decisions.

Wednesday, Time: 9.00-10.00, Room: Hall L1, Building L

The Infinite Models in Integer Programming

M. Conforti[1]

[1]University of Padova

The infinite models in integer programming were introduced by Gomory and Johnson and provide a template for the study of Integer Programs. In the past decade, there has been renewed interest in this approach We survey the most recent results and highlight the connections with other areas of mathematics.

Wednesday, Time: 16.00-17.00, Room: Hall L1, Building L

Choose your path wisely: iterative regularisation in a Bregman distance framework

C. B. Schönlieb[1]

[1]University of Cambridge

In this talk we present a generalisation of classical gradient descent that has become known in the literature as the so-called linearised Bregman iteration and — as the key novelty of this talk — apply it to minimise smooth but not necessarily convex objectives over a Banach space. We study the convergence of the proposed approach and present some numerical experiments for solving inverse imaging problems. This is joint work with Martin Benning, Marta Betcke and Matthias Ehrhardt.

Thursday, Time: 13.30-14.30, Room: Hall L1, Building L

Combinatorial Optimization under Nonlinear PDE-constraints

C. Buchheim[1] , R. Kuhlmann[2] , C. Meyer[1]

[1]Fakultät für Mathematik, Technische Universität Dortmund, [2]Zentrum für Technomathematik, Universität Bremen

We address optimal control problems containing semilinear elliptic PDE constraints as well as combinatorial constraints in the control variables, arising when modeling static diffusion processes controlled by discrete decisions. In the case of linear PDEs, such problems can be rewritten as (finite-dimensional) linear or quadratic integer programs. For the non-linear case, the standard solution approach is to directly discretize the entire problem, resulting however in huge non-convex mixed-integer optimization problems that can be solved to proven optimality only in very small dimensions. We propose a new approach based on outer approximation, using an integer linear programming master problem and a subproblem for calculating linear cutting planes. These cutting planes rely on the pointwise concavity and submodularity of the PDE solution operator in terms of the control variables, which we prove in the case of PDEs with a non-decreasing and convex nonlinear part. Our approach can handle general linear constraints on both control and state variables as well as tracking-type objective functions.

Thursday, Time: 14.30-15.30, Room: Hall L1, Building L

Shape Optimization for Free Boundary Problems: Analysis and Numerics

H. Harbrecht[1]

[1]Universitaet Basel

In this talk, the solution of Bernoulli type free boundary problems is considered by means of shape optimization. Different formulations are compared from an analytical and numerical point of view. By analyzing the shape Hessian in case of matching data, it is distinguished between well-posed and ill-posed formulations. A nonlinear Ritz–Galerkin method is applied for the discretization of the shape optimization problem. In case of well-posedness, existence and convergence of the approximate shapes is proven. Efficient first and second order shape optimization algorithms are obtained by means of modern boundary element methods.

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7. Minisymposia

MS 1. Linear algebra issues in optimization methods

Wednesday, Time: 10.30-12.30, Room: L1.202, Building L

Constraint-Preconditioned Krylov Methods for Regularized Saddle-Point Systems

D. di Serafino[1] , D. Orban[2]

[1]Department of Mathematics and Physics, University of Campania "L. Vanvitelli", Caserta,

[2]GERAD and Department of Mathematics and Industrial Engineering, École Polytechnique, Montréal

We are interested in the solution of regularized saddle-point systems, i.e., systems of the form

$$\begin{bmatrix} A & B^T \\ B & -C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ can be either symmetric or nonsymmetric, and $C \in \mathbb{R}^{m\times m}$ is symmetric positive semidefinite. These systems arise in many areas of scientific computing, e.g., in interior point and augmented Lagrangian methods for constrained optimization, and in stabilized finite-element discretizations of incompressible flow problems [1, 2]. In large-scale applications, the problem size often precludes the use of direct solvers, and iterative methods with suitable preconditioners must be applied. When A and C satisfy appropriate conditions, (1) can be solved using the conjugate gradient method coupled with a constraint preconditioner. Such preconditioners have proved widely effective, especially in optimization applications.

In this work, we investigate the design of constraint-preconditioned variants of Krylov methods for (1) by focusing on the underlying basis-generation process. We build upon [4] and exploit a reformulation of (1) given in [3] to provide general principles that allow us to specialize any Krylov method. In particular, we obtain constraint-preconditioned variants of Lanczos and Arnoldi-based methods, including MINRES, GMRES(m) and DQGMRES. We illustrate their numerical behaviour on several examples.

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Linear Algebra for Time-Dependent PDE-Constrained Optimization

J. Pearson[1]

[1]School of Mathematics, Statistics and Actuarial Science, University of Kent

A key application of linear algebra techniques within the field of mathematical optimization, as well as applied science more widely, is to PDE-constrained optimization problems. In this talk, we consider the development of fast and feasible preconditioned iterative methods for such problems, in particular those with a time-dependent component. To tackle the large and sparse matrix systems that arise from their discretization, we exploit the saddle point structure of the matrices involved, and develop potent approximations of the $(1, 1)$-block and Schur complement. Having motivated our general preconditioning strategy, we discuss applications of our method to problems with additional bound constraints on the PDE variables, formulations involving fractional derivatives within the constraints, and deferred correction techniques for time-dependent problems from fluid dynamics.

This talk forms part of the minisymposium "Linear algebra issues in optimization methods".

Douglas-Rachford iterations for two and three dimensional TV-, TGV- and Constrained TGV Denoising

L. Vestweber[1] , D. Lorenz[2] , H. Faßbender[1]

[1]TU Braunschweig, Institut Computational Mathematics, AG Numerik, [2]TU Braunschweig, Institut of Analysis and Algebra

We study the fully implicit primal-dual Douglas-Rachford splitting algorithm for denoising problems with TV- and TGV-penalty as well as constrained TGV. In each iteration one has to solve a large linear system of equations and we investigate the use of methods from numerical linear algebra to perform these solves efficiently. Especially we propose to use the preconditioned conjugate gradient method (pcg) with incomplete Cholesky factorization as a preconditioner. One can terminate the pcg iteration early after only a few iterations. The resulting method is competitive to existing methods and scales well to large problems.

Proximal splitting methods have become a workhorse in mathematical imaging. These methods split the objective function in an additive manner such that the proximal steps for the individual parts are simple to perform. If we consider the problem as a monotone inclusion with two monotone operators

$$0 \in \mathcal{A}(x) + \mathcal{B}(x)$$

proximal methods allow for *implicit* steps which apply the resolvents $\mathcal{R}_{t\mathcal{A}} = (I + t\mathcal{A})^{-1}$ and *explicit* steps where evaluation of the operator itself are used (provided they are single valued). Prominent examples are the forward-backward splitting [1] and its accelerated version [2], its adaption to saddle-point problems [5] or the Douglas-Rachford method [3, 4]. The latter is a fully implicit method and as such, does not have any step-size restriction to converge. For several monotone operators the resolvent is simple enough, but in other cases the evaluation of the resolvent can be a significant challenge.

We study simple total variation denoising as well as total generalized variation denoising in two and three space dimensions. In the latter case one can also work with constraints instead of penalty functionals. We formulate the respective optimization problem as a saddle point problem and apply the Douglas-Rachford method to an appropriate splitting of the respective optimality system similar to [3].

One of the resolvent steps needs a solution of a large linear system and we investigate tools from numerical analysis to perform this step efficiently. We observed that when employing an incomplete Cholesky decomposition as a preconditioner for the conjugate gradient method only two or just one iteration of pcg suffice to obtain convergence of the Douglas-Rachford method. In the case of only one iteration, we effectively apply a kind of preconditioned explicit step instead of a resolvent step, and a future research direction could be to investigate convergence in this case.

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Preconditioning Linear Systems from Deformable $3D$ Medical Image Registration using Low-rank Tensor Train format.

J. B. Nyenyezi[1] , A. Sartenaer[1]

[1] NaXys Institute, Namur University

In medical image analysis, deformable 3D image registration takes relatively long time. Especially for non-parametric transformations, the computing time may be quite troublesome and not even feasible for some clinical applications. Modeled as a variational problem, this registration needs to solve a sequence of linear systems during the optimization process. Much of the time is spent in the solution of these linear systems. Although these systems are often sparse and structured, they are very large and ill conditioned. Algorithms with linear complexity $\mathcal{O}(N)$ within Demons algorithms (this skips the linear systems [3]) are yet available. General reviews of these algorithms are available in the surveys within [4, 5] and references therein. However, this linear complexity is far too large for high-resolution and large $3D$ or $4D$ medical images.

To accelerate the registration process, the actual task is the development of algorithms which offer the most compromise between complexity and speed [4]. For this purpose, we aim to speed up this process with techniques that offer possibilities to replace a linear complexity $\mathcal{O}(n)$ by a quasi logarithmic complexity $\mathcal{O}(d \log n)$. This supposes the use of a compressed representation of data with a given accuracy ε $(0 \leq \varepsilon < 1)$. Certainly, this needs a good understanding of the acquisition process of the images and the application at hand. These last years, have been developed compression techniques for high-dimensional data using suitable data sparse representations [6]. In general, these representations known as numerical tensor representations are inspired by the variables separability, hierarchical matrix techniques and low-rank approximations. See [7, 8, 6]. They allow to transform linear systems in appropriate multilinear systems.

In this work we propose a Preconditioned Conjugate Gradient algorithm [9] to solve the multilinear system

$$\mathcal{M}\mathcal{A}\mathbf{x} = \mathcal{M}\mathbf{b}, \tag{1}$$

arising in deformable registration using the tensor train format. On one hand, the operator $\mathcal{A}$ is a Laplacian-like, for which it's possible to reorganize component's indices as products of new multi-indices that admit variables separability [1, 2]. In this way, the operator $\mathcal{A}$ is expressed as a sum of Kronecker products and the preconditioner $\mathcal{M}$ is a low-rank approximation of its inverse, computed via eigen elements. On the other hand, the right-hand $\mathbf{b}$ contains informations from the images. In addition to reorganizing indices for it's tensorization, the success of its low-rank approximation depends on the nature (modality) of the images and this infer to the quality of the low-rank solution $\mathbf{x}$. We have observed that algorithms using these techniques perform well for UltraSound (US), Computed Tomography (CT) and Positron Emission Tomography(PET) images , but are less good on Magnetic Resonance Images (MRI).

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MS 2. Numerical optimal control: Methods I

Tuesday, Time: 11.00-13.00, Room: Hall L1, Building L

Optimal control of a coupled system of hyperbolic and ordinary differential equations: FDTO and FOTD approaches and structure-exploiting SQP algorithms

S.-J. Kimmerle[1] , M. Gerdts[2]

[1]Institut für Mathematik und Bauinformatik (BAU-1), Universität der Bundeswehr München, [2]Institut für Mathematik und Rechneranwendung (LRT-1), Universität der Bundeswehr München

We consider the optimal control of a vehicle transporting a fluid container as load. The motion of the fluid is modelled by the Saint-Venant (shallow water) equations while the vehicle dynamics are described by Newton's equations of motion. The fluid container is mounted to the vehicle by a spring-damper element. The system may be controlled by the acceleration of the vehicle. This leads to an optimal control problem with a fully coupled system of nonlinear hyperbolic first-order partial differential equations (PDEs) and ordinary differential equations (ODEs).

We discuss different approaches for numerical optimal control of this system. For the time-optimal control with control and state constraints, a first-discretize-then-optimize approach has been pursued [1], using adjoint gradient computation. In order to reduce computing times, structure-exploiting optimisation algorithms of the sequential quadratic programming (SQP) type have been considered successfully [3] for the fully discretized problem. This approach is joint work with Jan-Hendrik Webert and Philip E. Gill. The idea of exploiting the particular structure may be applied to other optimal control problems as well. Furthermore, for our optimal control problem with fixed final time, a first-optimize-then-discretize approach (FOTD) has been examined as well [2]: we derive necessary optimality conditions rigorously and solve by a globalized semi-smooth Newton method in Hilbert spaces.

Finally, we discuss further examples for fully coupled systems of PDEs and ODEs and the corresponding optimal control problems.

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Group Sparsity in Optimal Control

C. Schneider[1] , G. Wachsmuth[2]

[1]Friedrich-Schiller-Universität Jena, Fakultät für Mathematik und Informatik,
[2]Technische Universität Chemnitz, Fakultät für Mathematik,

It is well known that optimal control problems with L^1-control costs produce sparse solutions, i.e., the optimal control is zero on whole intervals. In this talk, we study a general class of convex linear-quadratic optimal control problems with a sparsity functional that promotes a so-called *group sparsity* structure of the optimal controls. In this case, the components of the control function take the value zero on parts of the interval, simultaneously. These problems are both theoretically interesting and practically relevant. After obtaining results about the structure of the optimal controls, we derive stability estimates for the solution of the problem w.r.t. perturbations and L^2-regularization. These results are consequently applied to prove convergence of the Euler discretization. Finally, the usefulness of our approach is demonstrated by solving an illustrative example using a semismooth Newton method.

Structure-Preserving Discretization in Optimal Control

K. Flaßkamp[1] , C. Büskens[1]

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Optimal control problems arise in many fields of engineering, e.g. in robotics, space mission design, automotive or industrial engineering. Typically, there is an underlying mechanical system which is influenced by other system components, in particular, by the control unit. Optimal control searches for a control strategy that is admissible to the system dynamics, performs a given task, and minimizes a given cost criterion. Due to complex nonlinear dynamics, numerical techniques have to be applied in order to approximate optimal solutions. Direct methods based on a time-discretization of the dynamics have shown great applicability in real-word optimal control problems.

This talk focusses on the choice of time-discretization method in combination with non-linear optimization solvers for high-dimensional problems. Mechanical systems have characteristic properties such as their energy behavior, symmetries and symplecticity, which we aim to preserve in numerical optimal control (cf. [3]). Therefore, structure-preserving, so called symplectic integration methods are used (cf. [1]). Symplectic integrators can be derived by two approaches. Discretization of the variational principle leads to discrete-time forced Euler-Lagrange equations that define variational integrators (VI). Alternatively, symplectic integration methods can be designed by partitioned Runge-Kutta schemes for the configuration-momentum representation of mechanical systems (cf. [2]). In the optimal control problem, this leads to different choices of optimization variables and corresponding sparsity structures which influence the performance of the algorithm.

Previous research has mainly focussed on the benefits of VI discretization for the preservation of mechanical system structures. However, the optimal control problem additionally defines a Hamiltonian structure of state and adjoint variables. We numerically study the interplay between structure-preserving discretization methods for state and adjoint systems.

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Optimal Vaccination Strategies against Dengue Fever

K. Chudej[1] , A. Fischer[2]

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Dengue fever is a virus infection transmitted by mosquitos which affects more than half of the world's population. According to WHO calculations, every year up to 100 million infections with the vector-borne disease can be registered, thereof half a million haemorrhagic infections and 22 000 deaths. Only since the 1980s serious epidemics occured and in the last 25 years the epidemic potential of the virus has increased. Symptoms of the virus include high fever, headache and rash while the haemorrhagic variant can cause bleedings from mouth and nose, severe stomachache and continuous vomiting which lead to a leakage of blood and plasma that can result in organ failure and shock. Further, dengue fever is the leading cause for hospitalisation and infant mortality in many southeast Asian countries. The virus is transmitted to human beings by the bite of an infectious Asian Tiger mosquito, especially those of the species *Aedes Aegypti* and *Aedes Albopictus*. The vectors, which become infected by biting an infectious human, require the blood to hatch their eggs. Since the mosquitos prefer to breed in stale water, left used tires, flower pots and even swimming pools offer ideal circumstances for the reproduction of the population. Due to their ability to adapt to new conditions rapidly, international tourism and trade continue to enlarge their habitat.

Upto now all dengue cases in Germany could be traced back to an infection outside of Germany. The historical spread of the vector populations and climatic analyses indicate a rising probability of a dengue fever outbreak in Europe. Until World War II mosquitos could be found in regions of Europe with mild climatic conditions and only vanished due to the spraying of DDT and a change of agriculture. However, in the past decades, *Aedes Albopictus* has been one of the fastest spreading species worldwide. Particularly the international trade of plants and used tires caused an uncontrolled import of mosquito larvae into various regions of the world. In Europe since the 1990s mosquito populations can be found in France, Italy and Switzerland. Additionally, against prior assumptions, newer studies demonstrate the ability of the species' eggs to survive temperatures as low as $-10°C$. In 2014, researchers in Freiburg im Breisgau were able to collect specimens in various breeding places whose total number indicated the existence of a sufficient amount of mosquitos to form a survivable population. Based on the mentioned observations, the upper Rhine Valley, having relatively mild climatic conditions and a major transport axis from the south, offers ideal requirements for the vectors in Germany. Thus, climatic warming and the adaptability of the mosquitos imply a possible spread of a vector population in Freiburg im Breisgau and as a result the threat of infectious diseases.

Due to the fact that at the end of 2015 a vaccine againt dengue fever was licensed, the control of the virus infection is no longer merely based on mosquito control and the avoidance of vector to human contact. Thus, this talk examines the effects of various control strategies including vector control and vaccination campaigns, using a system of ordinary differential equations.

MS 3. Numerical optimal control: Methods II

Wednesday, Time: 10.30-12.30, Room: Hall L1, Building L

On the Approximation of Infinite Horizon Optimal Control Problems

K. Worthmann[1]

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We consider a stabilization task for a nonholonomic mobile robot with system dynamics

$$\dot{x}(t) = \begin{pmatrix} \cos\, x_3(t) \\ \sin\, x_3(t) \\ 0 \end{pmatrix} u_1(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_2(t). \tag{1}$$

Control and state constraints are modelled by $\mathbb{U} \subset \mathbb{R}^2$ and $\mathbb{X} \subset \mathbb{R}^3$, respectively. Both sets $\mathbb{X}$ and $\mathbb{U}$ are convex, compact, and contain the origin in its interior. To tackle this problem, we set up the following optimal control problem:

$$\min_{u \in \mathcal{L}^1_{\mathrm{loc}}([0,\infty),\mathbb{U})} \int_0^\infty \ell(x(t), u(t))\, \mathrm{d}t \qquad \text{subject to} \qquad (1) \text{ and } x(t) \in \mathbb{X} \text{ for all } t \geq 0$$

with running costs $\ell : \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ satisfying $\ell(x,u) = 0$ if and only if $(x,u) = (0,0)$. Since a direct solution is, in general, computationally intractable, we want to apply Model Predictive Control (MPC), i.e., we replace the original infinite horizon problem by an iteratively solved sequence of optimal control problems on a finite horizon, see [1] for details on MPC.

In this talk, we first explain, why quadratic running costs ℓ do not work in MPC without additional terminal costs or constraints [2]. Moreover, we show that even a potentially large terminal weight does not resolve this issue. Then, we give details on the design of suitable running costs ℓ, for which asymptotic stability of the MPC closed loop can be rigorously shown, see [4, 3]. Finally, we illustrate our findings by numerical simulations.

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On Model Predictive Control for Probability Density Functions

A. Fleig[1] , L. Grüne[1] , R. Guglielmi[2]

[1]Department of Mathematics, University of Bayreuth, [2]Dyrecta Lab

Optimal control of a stochastic process that is modeled by an Itô stochastic differential equation can often be achieved via controlling the underlying probability density function (PDF). Its time evolution is prescribed by the Fokker-Planck equation, a second order parabolic partial differential equation (PDE). In this manner, the original stochastic optimal control problem can be reformulated as a deterministic PDE-constrained optimization problem. A Model Predictive Control (MPC) scheme is then applied to track the PDF over a fixed time horizon.

In this talk, we analyze the optimal control problem subject to the Fokker-Planck equation. As in [2], we consider stochastic processes that are controlled through the drift term. In this case, the control appears as a coefficient of the advection term in the Fokker-Planck equation, resulting in a bilinear optimal control problem. In contrast to [2], the control function may depend on space (and time). We present results regarding the existence of optimal controls and derive the first order necessary optimality conditions for common L^2 cost functionals of tracking type [3].

Furthermore, we study the stability of the MPC closed loop feedback system, where we avoid stabilizing terminal costs or constraints. In this case, the MPC horizon length plays a crucial role. Similar to [1], we apply a technique based on the exponential controllability condition to find a (minimal) stabilizing horizon length. In this context, the controlled Ornstein-Uhlenbeck process, with a control that is time-dependent and either constant or linear in space, is of particular interest. Numerical simulations complement the analysis.

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A Dual Based Pseudospectral Method for Infinite Horizon Optimal Control Problems

K. Kolo[1] , S. Pickenhain[1]

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We consider a class of infinite horizon optimal control problems with vector-valued states and controls involving the Lebesgue integral in the objective and a dynamics linear with respect the control.

This special class of problems arises in the theory of economic growth, epidemic models and in processes where the time T is an exponentially distributed random variable.

The problem is formulated as an optimization problem in Hilbert Spaces. The remarkable on this statement is the choice of Weighted Sobolev- and Weighted Lebesgue spaces as state and control spaces respectively.

These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a Hilbert space.

For the class of problems proposed, we are able to derive a related dual program in form of a variational problem in Hilbert Spaces by using the integrated Hamiltonian defect and formulate a Pseudo Maximum Principle for these problems.

Based on these principles we use an indirect pseudospectral method introduced in [1], to solve the problem numerically. Numerical results are presented for a quadratic regulator model.

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MS 4. Numerical optimal control with PDE constraints

Monday, Time: 15.30-17.30, Room: Hall L1, Building L

Gradient Descent Methods for Optimization with Mixed-Integer and PDE Constraints

F. M. Hante[1]

[1]Department Mathematik, Universität Erlangen-Nürnberg

Motivated by challenging tasks such as minimal cost operation of gas, water or traffic networks we consider mixed-integer optimization problems subject to constraints in form of evolution type partial differential equations. The integer restrictions model discrete control components, for example valves, gates or signal signs in the mentioned applications. We consider relaxation and reparameterization techniques in order to solve such problems in an appropriate sense locally using gradient descent methods. A particular focus is given to problems involving switching costs. The descent strategies are designed such that gradients of certain subproblems can be obtained numerically very efficiently by solutions of suitable adjoint problems. The performance of these methods will be demonstrated on numerical examples.

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A multigrid scheme for solving convection-diffusion-integral optimal control problems

D. Gathungu[1] , A. Borzì[1]

[1]Universität Würzburg

The fast multigrid solution of an optimal control problem governed by a convection-diffusion partial-integro differential equation is investigated. This optimization problem considers a cost functional of tracking type and a constrained distributed control. The optimal control sought is characterized by the solution to the corresponding optimality system, which is approximated by a finite volume and quadrature discretization schemes and solved by multigrid techniques. The proposed multigrid approach combines a multigrid method for the governing model with a fast multigrid integration method. The convergence of the proposed solution procedure is analyzed by local Fourier analysis and validated by results of numerical experiments.

A numerical solver for the Fokker-Planck optimal control of stochastic jump-diffusion processes

M. Annunziato[1]

[1]Università degli Studi di Salerno

The problem to calculate the optimal control of a jump-diffusion stochastic processes is tackled in the framework of the Fokker-Planck control [1, 3, 4]. The Fokker-Planck equation (FPE), governing the evolution of the time dependent probability density function of the process, has the form of a partial integro-differential equation (PIDE).

The control function acts on a parameter of the FPE, and its objective is the minimization of a tracking functional and the cost of the control. The latter is evaluated with the norms L2 , and L1 in order to promote the sparsity of the control. Furthermore, since the process is bounded from barriers, appropriate boundary conditions are included in the formulation of the optimization problem. The minimization problem is formulated in terms of a first order necessary optimality system, composed of a pair (forward-backward) of PIDEs and the optimality condition. The PIDEs are solved with a numerical method that uses the Strang-Marchuck splitting [6], i.e. by evaluating the differential and the integral operators in two separate stages, one implicit the other explicit. The spatial operator of the first is approximated with the Chang-Cooper method [7], the second with the mid-point quadrature rule. The complete numerical scheme is proved to be second order accurate, conservative, positive, stable and convergent in the L2 norm under Courant- Friedrichs-Lewy-like conditions [2]. Moreover, in the case of non-smooth L1 cost, a "proximal" iterative method [1, 5] for the minimization of the cost is used. Numerical experiments validates the ability to achieve the control of the jump-diffusion process. The author acknowledges financial support from Istituto Nazionale di Alta Matematica "F. Severi", sezione GNCS, Città Universitaria - P.le Aldo Moro 5, 00185 - Roma, Italy.

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MS 5. Numerical optimal control: Applications

Thursday, Time: 8.00-10.00, Room: Hall L1, Building L

Optimal Control of a Global Model of Climate Change with Adaptation and Mitigation

H. Maurer[1] , M. Atolia[2] , P. Loungani[3] , W. Semmler[4]

[1]Westfälische Wilhelms-Universität Münster, Angewandte Mathematik: Institut für Analysis und Numerik, [2]Department of Economics, Florida State University [3]Research Department, International Monetary Fund, [4]Department of Economics, New School of Research and University of Bielefeld

The Paris 2015 agreement on climate change is aiming at reducing the temperature increase to below $2^o\,C$. This implies that effective mitigation policies need to be pursued that not only prevent the CO_2 emission from rising further but to reduce the annual emission substantially. The modeling strategy we pursue here, thus, attempts to answer three questions: First, what are the best strategies to keep the CO_2 emission bounded by a predefined upper bound. Second, what resources should be allocated to the adaptation effort when climate risk, due to a lack of emission reduction, is rising and future economic, social, and ecological damages can be expected. A third issue is of how the efforts of mitigation and adaptation are funded and how the funds should efficiently be allocated between traditional infrastructure investment, mitigation and adaptation efforts.

We have developed a dynamic global model with 5 state and 5 control variables that allows to consider the specific policies of infrastructure investment, mitigation and adaptation. There are two sources of energy: non-renewable (brown) energy produced by an extractive resource sector and renewable (green) energy produced with private physical and green capital. The emissions from brown energy use are a source of negative externality that directly enters the (instantaneous) welfare functional. A numerical challenge arises from the fact that the optimal control model involves a nonlinear mixed control-state constraint. The optimal control model is an extension of the models presented in [1], [2]. We discuss the necessary optimality conditions and determine stationary points of the canonical system. Discretization and nonlinear programming methods then allow us to determine optimal control policies for various initial conditions and terminal constraints.

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Parameter Identification for Robotic Systems Using Optimal Control Techniques

K. Schäfer[1] , M. Runge[1] , K. Flaßkamp[1] , C. Büskens[1]

[1]Center for Industrial Mathematics, WG Optimization and Optimal Control, University of Bremen

In many engineering applications it is desired to find a suitable model which sufficiently describes the dynamic behavior of the system. The identification of physical as well as non-physical parameters within this model is an important task for which numerous experimental and numerical methods exist.

In order to model the dynamic behavior of an industrial robot we seek to numerically identify (physical) parameters arising in the corresponding Euler-Lagrange equations (for a derivation see e.g. [3]) by solving parameter identification problems. The general aim is to fit the model to real data measured by sensors, in particular to find a set of parameters for which the resulting state trajectories approximate a given motion best. This leads to a least squares approach in the objective function. Thus, the problem of parameter identification can be formulated as an optimization problem with a system of ordinary differential equations as equality constraints, which motivates the usage of optimal control techniques.

In this work we focus on the comparison of different direct optimal control approaches, namely a reduced discretization against a full discretization method. In [2] it is stated that the technique of multiple shooting possesses advantages over the single shooting approach. We compare the methods with respect to various criteria, e.g. robustness, efficiency or the quality of the calculated set of parameters and state trajectories. By restricting to a simplified model of an industrial robot having two degrees of freedom we demonstrate the effects of the different approaches, while the obtained results are validated on a much more complex industrial robot (DENSO VS-050, six links).

After discretization, the formulation of the parameter identification problem as an optimal control problem leads to a (possibly large-scale) nonlinear optimization problem. We use the software package TransWORHP to transcribe the optimal control problem into a nonlinear optimization problem. To find a local optimum, it uses the NLP solver WORHP (see [1]), which is able to exploit structural sparsity information.

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The Dubins Car Problem with Delay and Applications to Aeronautics Motion Planning Problems

R. Bonalli[1] , B. Hérissé[1] , H. Maurer[2] , E. Trélat[3]

[1]Onera - The French Aerospace Lab, [2]Institut für Numerische Mathematik Westfälische Wilhelms-Universität Münster, [3]Sorbonne Universités, Laboratoire Jacques-Louis Lions

Many optimal control problems involve delays on the state and/or on the control, in particular in motion planning. In launch vehicle applications (e.g. atmospheric reentry, coplanar orbit transfer, rendezvous problems, etc.), the so called *non-minimum phase problem* [1] plays an important role and it can be modelled as a delay within the rotational dynamics of the vehicle.

In order to give a detailed behaviour of solutions of 2D rendezvous problems considering non-minimum phase, we introduce and analyse the following new class of optimal control problems

$$(P_\tau)\begin{cases}\min \quad g(T, r(T)) \quad , \quad T \text{ free} \\ \dot{x}(t) = v(t)\cos\theta(t) \quad , \quad \dot{y}(t) = v(t)\sin\theta(t) \quad , \quad \dot{\theta}(t) = \frac{\delta(t)+\delta(t-\tau)}{2} \\ \dot{v}(t) = -C\delta^2(t) \quad , \quad \dot{\delta}(t) = u(t) \quad , \quad |u(t)| \leq u_{\max} \quad , \quad \tau \ , \ C > 0\end{cases}$$

where the state is $r = (x, y, \theta, v, \delta)$. Problem (P_τ) can be seen as a generalization of the well-known *Dubins car problem*, in which, a delay within the rotational dynamics is added, and moreover, a general Mayer cost is considered.

Under appropriate assumptions on $g(T, r(T))$, we prove that a discontinuity occurs at $T - \tau$ in the singular part of the optimal control of (P_τ) (Figure 1). The same structure is found for the 2D rendezvous with non-minimum phase.
We develop two numerical approaches to solve the previous problems: direct methods [2] and shootings coupled with homotopy methods [3]. We will discuss the pros and cons of these approaches.

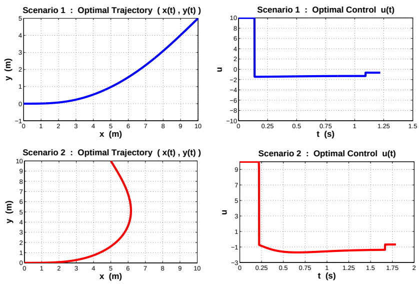

Figure 1: Optimal Solutions for Different Scenarios of (P_τ).

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MS 6. Methods for continuous optimization and applications

Thursday, Time: 8.00-10.00, Room: L1.202, Building L

Modified Augmented Lagrangian Methods in Finite and Infinite Dimensions

C. Kanzow[1]

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The classical augmented Lagrangian method belongs to the standard approaches for the solution of constrained optimization problems. Recent modifications by Andreani, Birgin, Martínez and Co-workers for finite-dimensional optimization problems turn out to have stronger properties than the classical method. In this talk, we begin with a review of these classical and modified augmented Lagrangian methods. We then present an extension of the modified method to optimization problems in Banach spaces. Some numerical results illustrate the reliability of the proposed technique.

The talk is based on joint work with Daniel Steck and Daniel Wachsmuth.

This research was supported by the German Research Foundation (DFG) within the priority program "Non-smooth and Complementarity-based Distributed Parameter Systems: Simulation and Hierarchical Optimization" (SPP 1962) under grant number KA 1296/24-1.

Distributed Newton-like Methods with Variable Number of Working Nodes

N. Krklec Jerinkić[1] , D. Bajović[2] , D. Jakovetić[1] , N. Krejić[1]

[1]Faculty of Sciences, University of Novi Sad, [2]Faculty of Technical Sciences, University of Novi Sad

Recently, an idling mechanism has been introduced in the context of distributed first order methods for minimization of a sum of nodes' local convex costs over a generic, connected network. The idling mechanism operates in such a way that an increasing number of nodes becomes involved in the algorithm (on average) as the iteration counter k grows, thus avoiding unnecessarily expensive exact updates at the initial iterations while performing beneficial close-to-exact updates near the solution. Here, we present a methodology that demonstrates how idling can be successfully incorporated in distributed second order methods also. Interestingly, a second order method with idling exhibits very similar theoretical convergence and convergence rates properties as the corresponding standard method (without idling), but with significantly cheaper updates. This usually results in significant communication and computational savings of the idling-based method.

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Quasi-Newton methods for Constrained Nonlinear Systems and Application to Gas Distribution Networks

M. Porcelli[1] , B. Morini[1]

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We address the solution of convex constrained nonlinear systems by new linesearch Quasi-Newton methods. These methods are based on a proper use of the projection map onto the constraint set and on a derivative-free and nonmonotone linesearch strategy. The convergence properties of the proposed methods are presented along with a worst-case iteration complexity bound. Several implementations of the proposed scheme are discussed and validated on bound-constrained problems including gas distribution network models.

Newton's Method for the Primal Training of Support Vector Machines

N. Strasdat[1] , A. Fischer[1] , K. Luig[2] , T. Thies[2] , F. Weber[2]

[1]Technische Universität Dresden, [2]Cognitec Systems GmbH

Support Vector Machines (SVMs) are a basic tool for machine learning. They are used in many applications to handle classification and regression problems. In order to take advantage of large datasets, efficient training methods are necessary. In the original formulation of the training problem, the objective is to minimize a convex function over some (possibly infinite dimensional) inner product space. It is well-known that this optimization problem can be reformulated as a finite-dimensional convex problem. However, this approach involves ambiguity of the solution in general. We present a characterization of this non-uniqueness. Moreover, we apply the theoretical insight to construct an adapted version of Newton's method for the efficient solution of SVM training problems.

MS 7. Optimization in finance

Thursday, Time: 10.30-12.30, Room: L1.202, Building L

Bregman iteration for portfolio selection

S. Corsaro[1] , Z. Marino[1]

[1]University of Naples Parthenope

We focus on the the portfolio selection problem. In the single-period case, we consider the l_1 -regularized Markowitz model, where a l_1-penalty term is added to the objective function of the classical mean-variance one to stabilize the solution process, promoting sparsity in the solution and avoiding short positions [1]. We consider the Bregman iteration method to solve the related constrained optimization problem. We propose

an iterative algorithm based on a modified Bregman iteration, in which an adaptive updating rule for the regularization parameter is defined; the modified scheme preserves the properties of the original one [2]. We then show extension of the approach to the multi-period case.

References

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Optimization meets diversification for portfolio selection

F. Tardella[1]

[1]Sapienza University of Rome

The classical approach to portfolio selection calls for finding a feasible portfolio that optimizes one of the several proposed risk measures, or (expected) utility functions, or performance indexes. However, the optimization approach might be misleading due to the difficulty of obtaining good estimates for the parameters involved in the function to be optimized and to the high sensitivity of the optimal solutions to the input data.

This observation has led some researchers to claim that a straightforward capital diversification, i.e., the Equally Weighted portfolio can hardly be beaten by an optimized portfolio [3]. However, if the market contains assets with very different intrinsic risks, then this leads to a portfolio with limited total risk diversification. Therefore, alternative risk diversification approaches to portfolio selection have been proposed, such as the practitioners' approach of taking weights proportional to $1/\sigma_i$, where σ_i is the volatility of asset i. A more thorough approach to risk diversification requires to formalize the notion of *risk contribution* of each asset, and then to manage it by a model. For example the *Risk Parity* approach (see [5], and references therein) aims at a portfolio where the total risk contributions of all assets are equal among them [4]. The original risk parity approach was applied to volatility. However alternative risk measures can also be considered (see, e.g., [1]). It can also be shown that the Risk Parity approach is actually dominated by *Equal Risk Bounding* [2], where the total risk contributions of all assets are bounded by a common threshold which is then minimized. Furthermore, several alternative approaches to diversify risk have recently appeared in the literature.

We propose here a new approach that tries to reduce the impact of data estimation errors and to join the benefits of the optimization and of the diversification approaches by choosing the portfolio that is best diversified (e.g., Equally Weighted or Risk Parity) on a subset of assets of the market, and that optimizes an appropriate risk, or utility, or performance measure among all portfolios of this type. We show that this approach yields portfolios that are only slightly suboptimal in-sample, and generally show improved out-of-sample performance with respect to their purely diversified or purely optimized counterparts.

Keywords: Portfolio selection, risk diversification, pseudoBoolean optimization

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Heuristics for Portfolio Selection

E. Schumann[1]

[1]CPV/CAP, Switzerland

Heuristic optimisation methods are contentious. They offer 'good' solutions to optimisation models that would otherwise be difficult or impossible to solve. On the other hand, they are not guaranteed to provide the truly-optimal solution; they only give us a stochastic approximation of the optimum. I discuss the application of heuristics to portfolio selection, with emphasis on two statements: i) heuristics provide indeed 'good' solutions to models, and ii) 'good' solutions are sufficient for financial models. As evidence for i), I present results of an application of local-search techniques to equity-portfolio construction and demonstrate the benefits of models that could only be solved with heuristics. As evidence for ii), I demonstrate that the randomness of the 'optimal' solution obtained from a heuristic can be made so small that for all practical purposes it can be neglected. Any further in-sample improvement will, out-of-sample, only lead to financially meaningless improvements or unpredictable changes (noise) in performance.

Trading regions and structure of optimal portfolio policies under transaction costs

J. Sass[1]

[1]Department of Mathematics, University of Kaiserslautern, Germany

In a Black Scholes financial market - consisting of one stock whose prices evolve like a geometric Brownian motion and one risk free-asset - an investor wants to maximize, say, the growth rate of her/his portfolio (wealth). Without transaction costs the optimal policy would be given by the constant Merton fraction which is the fraction of wealth to be invested in the stock. Facing transaction costs it is no longer adequate to keep this risky fraction constant. Trading regions will arise, where it is optimal to buy stocks, to sell stocks, or not to trade at all.

In this talk we discuss the shape of optimal trading regions for different types of transaction costs, model assumptions and optimization criteria. We further discuss the extension to more stocks and numerical procedure to find the trading regions. In

the one-dimensional case we analyze the structure of the optimal strategies in detail.

MS 8. Variational problems and their interdependences

MS 8a. Part I

Tuesday, Time: 11.00-13.00, Room: L1.201, Building L

Newton-type Methods for Fritz John Systems of Generalized Nash Equilibrium Problems

A. Fischer[1] , M. Herrich[1]

[1]Institute of Numerical Mathematics, Technische Universität Dresden

A well-known approach to solve a generalized Nash equilibrium problem (GNEP) is to consider a necessary optimality condition and to reformulate it as a nonsmooth system of equations. Frequently, the Karush-Kuhn-Tucker (KKT) conditions of all players are concatenated. It was shown [1] that, due to the lack of a suitable constraint qualification, solutions of a GNEP may exist that do not satisfy the KKT but the Fritz John (FJ) conditions. The corresponding nonsmooth system of equations is (similar to the KKT case) underdetermined since we assume that there are constraints shared by all players. We show that some Newton-type methods [2, 3] recently developed for certain constrained systems of nonsmooth equations can be successfully applied to a nonsmooth system that is equivalent to the FJ conditions for GNEPs. In particular, we provide conditions for local quadratic convergence which are weaker than existing ones.

This presentation is part of the minisymposium "Variational problems and their interdependences".

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Generalized Nash Equilibria and their Computation via Augmented Lagrangian Methods

D. Steck[1] , C. Kanzow[1] , D. Wachsmuth[1] , V. Karl[1]

[1]University of Würzburg

The augmented Lagrangian method is a classical technique for nonlinear programming which has been extended to generalized Nash equilibrium problems (GNEPs) and even quasi-variational inequalities. In this talk, we present an augmented Lagrangian

method for finite-dimensional GNEPs, describe some key ingredients for its convergence analysis, and discuss possible interpretations of our assumptions and results. We then proceed by briefly discussing GNEPs in infinite dimensions. In particular, we give a simple existence result for normalized Nash equilibria, and describe how augmented Lagrangian methods can be applied to this setting.

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A bridge between bilevel programs and Nash games

S. Sagratella[1] , L. Lampariello[2]

[1]Department of Computer, Control and Management Engineering Antonio Ruberti, Sapienza University of Rome, [2]Department of Business Studies, Roma Tre University

We study connections between optimistic bilevel programs and generalized Nash equilibrium problems [1]. Namely, in a multi-agent framework, we consider both the "vertical case", in which there is a leader that can anticipate the strategies of a follower (bilevel problem), and the "horizontal case", in which both the agents must decide their strategies simultaneously (Nash game). We propose a new "uneven horizontal case" in which the agents play a Nash game that incorporates some taste of hierarchy. We define classes of problems for which solutions of the bilevel problem can be computed by finding equilibria of the uneven Nash game. Our study provides the theoretical backbone and the main ideas underlying some useful novel algorithmic developments. Exploiting these results, we tackle some classes of multi-leader-follower games stemmed from electricity markets.

This presentation is part of the minisymposium "Variational problems and their interdependences".

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MS 8b. Part II

Thursday, Time: 10.30-12.30, Room: L1.201, Building L

Regularization for a Complementarity Formulation of Cardinality Constrained Optimization Problems

M. Bucher[1] , A. Schwartz[1]

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Cardinality constraints are used to model the fact that the solution of an optimization problem is expected or desired to be sparse. They impose an upper bound on the cardinality of the support of feasible points. A possibly nonlinear objective function is to be minimized subject to the cardinality constraint as well as further nonlinear constraints. Applications of cardinality constraints include portfolio optimization, compressed sensing or the subset selection problem in regression.

Cardinality constrained optimization problems are hard to solve, since the mapping of a vector to the cardinality of its support is a discontinuous function. Even testing feasibility of a vector is known to be NP-hard [1]. Solution techniques for cardinality constrained optimization problems include the reformulation as a mixed-integer nonlinear program or the approximation of the cardinality constraint with a ℓ^1-norm.

We follow an approach by Burdakov, Kanzow and Schwartz [2] which is a reformulation of the cardinality constraint with complementarity constraints using continuous auxiliary variables. This opens up the possibility to use methods from nonlinear optimization. The reformulation possesses a strong similarity to a mathematical program with complementarity constraints (MPCC) and, like an MPCC, does not fulfil standard constraint qualifications. We use the strong link between the aforementioned reformulation of cardinality constrained optimization problems and MPCCs to derive optimality conditions and numerical methods. Particularly we investigate second order optimality conditions for the reformulation. We then use these to derive new convergence results for a Scholtes-type regularization. We compare this regularization to other methods and present numerical results.

The work of Max Bucher and Alexandra Schwartz This work is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt Darmstadt.

This presentation is part of the minisymposium "Variational problems and their interdependences".

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Generalized Nash equilibria and semi-infinite programming

G. Bigi[1] , S. Sagratella[2]

[1]Dipartimento di Informatica, Università di Pisa, [2]Dipartimento di Ingegneria Informatica, Automatica e Gestionale, Sapienza Università di Roma

Generalized Nash games (shortly GNEPs) and semi-infinite programs (shortly SIPs) share some similarities that lead to meaningful connections. In this talk we analyse these connections and we show how algorithms for GNEPs can be exploited to solve

SIPs. Indeed, SIPs can be reformulated as GNEPs with a peculiar structure under some mild assumptions. Pairing this structure with the partial penalization scheme for GNEPs developed in [2] leads to a class of solution methods for SIPs that are based on a sequence of saddlepoint problems. Any converging algorithm for the saddlepoint problem provides the basic iterations to perform the penalty updating scheme. In particular, a projected subgradient method for nonsmooth optimization (see [1]) and a subgradient method for saddlepoints [3] are adapted to our framework and the convergence of the resulting algorithms is shown.

This presentation is part of the minisymposium "Variational problems and their interdependences".

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MS 9. Model order reduction in PDE-constrained optimization

Thursday, Time: 10.30-12.30, Room: Hall L1, Building L

Reduced Order Modeling for Time-Dependent Optimal Control Problems with Variable Initial Values

D. Jando[1]

[1]Interdisciplinary Center for Scientific Computing, Heidelberg University

In this talk I will present a new reduced order model (ROM) Hessian approximation for large-scale linear-quadratic optimal control problems (OCPs) with variable initial values. Such problems arise, for example, as subproblems in multiple shooting formulations of OCPs constrained by instationary PDEs. Apart from the controls also the intermediate initial values introduced by multiple shooting are optimization variables.

The computation of a Hessian vector product requires the solution of the linearized state equation for the vector of controls and initial data to which the Hessian is applied to, followed by the solution of the second order adjoint equation. To speed up computations, projection based ROMs of these two equations are used to generate Hessian approximations for the state variables. The challenge is that in general no fixed ROM well-approximates neither the application of the Hessian to all possible initial values nor the solution of the corresponding linear equation for all possible right hand sides. Our recently proposed approach, after having selected a basic ROM, augments this basic ROM by one vector which is either the right hand side or the initial value. This augmentation improves the ROM quality significantly. We use these Hessian approximations in a conjugate gradient method to approximate the optimal initial value. Although the augmented ROM substantially improves the accuracy of the computed initial value, this accuracy may still not be enough. Thus, I will also present a new sequential approach based on the ROM augmentation which allows to compute an approximate initial value with the same accuracy as the one obtained using the expensive full order model, but at a fraction of the cost. Finally, we will come back to the problem with control variables and apply the presented ROM approaches to efficiently solve this kind of problems.

Reduced basis approximation and a posteriori error bounds for 4D-Var data assimilation

M. Grepl[1] , S. Boyaval[2] , M. Kärcher[3] , K. Veroy[3]

[1]Numerical Mathematics (IGPM), RWTH Aachen, [2]Laboratoire d'hydraulique Saint-Venant, Université Paris-Est, [3]Institute for Advanced Study in Computational Engineering Science, RWTH Aachen

The reduced basis method is a certified model order reduction technique for the rapid and reliable solution of parametrised partial differential equations, and it is especially suited for the many-query, real-time, and slim computing contexts. In this talk, we focus on problems in optimal control and data assimilation. In particular, we present a certified RB approach to four dimensional variational data assimilation (4D-Var) [2, 3].

Several contributions have explored the use of reduced order models as surrogates in a 4D-Var setting (see, for instance, [1, 4, 6]). We consider the particular case in which the behaviour of the system is modelled by a parametrised parabolic partial differential equation where the initial condition and model parameters (e.g., material or geometric properties) are unknown, and where the model itself may be imperfect. We consider (i) the standard strong-constraint 4D-Var approach, which uses the given observational data to estimate the unknown initial condition of the model, and (ii) the weak-constraint 4D-Var formulation, which additionally provides an estimate for the model error, and thus can deal with imperfect models [5]. Since the model error is a distributed function in both space and time, the 4D-Var formulation generally leads to a large-scale optimization problem that must be solved for every given parameter instance. We introduce reduced basis spaces for the state, adjoint, initial condition, and model error. We then build upon recent results on RB methods for optimal control problems in order to derive *a posteriori* error estimates for reduced basis approximations to solutions of the 4D-Var problem. Numerical tests are conducted to verify the validity of the proposed approach.

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Greedy Controllability of Reduced-Order Linear Dynamical Systems

L. Iapichino[1] , G. Fabrini[2] , S. Volkwein[2]

[1]Eindhoven University of Technology, [2]University of Konstanz

Often a dynamical system is characterized by one or more parameters describing physical features of the problem or geometrical configurations of the computational domain. As a consequence, by assuming that the system is controllable, corresponding to different parameter values, a range of optimal controls exists. The goal of the proposed approach is to avoid the computation of a control function for any instance of the parameters. The greedy controllability [1] consists in the selection of the most representative values of the parameters that allows a rapid approximation of the control function for any desired new parameter value, ensuring that the system is steered to the target within a certain accuracy. By proposing the Reduced Basis method [2] (an efficient model order reduction technique) in this framework, the computational costs are drastically reduced and the efficiency of the greedy controllability approach is significantly improved.

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Adaptive trust-region POD for optimal control of the Cahn-Hilliard equation

N. Scharmacher[1] , C. Gräßle[1] , M. Hinze[1]

[1]Universität Hamburg

We consider the optimal control of a Cahn-Hilliard system in a trust-region framework. For an efficient numerical solution, the expensive high dimensional PDE systems are replaced by reduced order models utilizing proper orthogonal decomposition (POD-ROM). Within the trust- region POD (TR-POD), the accuracy of the surrogate models is controlled in the course of the optimization. The POD modes are computed corresponding to snapshots of the governing equations which are discretized utilizing adaptive finite elements. In the numerical examples, the smooth as well as the double-obstacle free energy potential are considered.

MS 10. New hierarchies of SDP relaxations for polynomial systems

Wednesday, Time: 10.30-12.30, Room: L1.201, Building L

Interval Enclosures of Upper Bounds of Roundoff Errors using Semidefinite Programming

V. Magron[1]

[1]VERIMAG-CNRS

A longstanding problem related to floating-point implementation of numerical programs is to provide efficient yet precise analysis of output errors. We present a framework to compute lower bounds of absolute roundoff errors, for a particular rounding model. This method ap- plies for numerical programs implementing polynomial functions with box constrained input variables. Our study relies on semidefinite programming relaxations and is complementary of over-approximation frameworks, consisting of obtaining upper bounds for the absolute roundoff error. Combining the results of both frameworks allows to get interval enclosures for upper bounds of roundoff errors. The recent over-approximation framework [2], related to the `Real2Float` software package, em- ploys semidefinite programming to obtain a hierarchy of upper bounds converging to the abso- lute roundoff error. This hierarchy is derived from the general moment-sum-of-squares hierarchy (also called Lasserre's hierarchy) initially provided in the context of polynomial optimization. While this first semidefinite hierarchy allows to approximate from above the maximum of a polynomial, [1] provides a complementary semidefinite hierarchy, yielding this time a sequence of converging lower bounds. Our under-approximation framework is based on a new hierarchy of convergent robust semidef- inite approximations for certain classes of polynomial optimization problems. Each problem in this hierarchy can be exactly solved via semidefinite programming.

By using this hierarchy, one can provide a monotone non-decreasing sequence of lower bounds converging to the abso- lute roundoff error of a program implementing a polynomial function, applying for a particular rounding model. We release a software package called `FPSDP` implementing this framework. We investigate the efficiency and precision of our method by comparing the performance of `FPSDP` with existing tools on non-trivial polynomial programs coming from space control, optimization and computational biology.

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Semidefinite Characterization of Invariant Measures for Polynomial Systems

M. Forets[1] , D. Henrion[2] , V. Magron[1]

[1]VERIMAG-CNRS, [2]LAAS-CNRS

We consider the problem of characterizing measures which are invariant with respect to the dynamics of either discrete-time or continuous-time polynomial systems, under general semial- gebraic set constraints. First, we address the problem of approximating the density of invariant measures which are absolutely continuous with respect to the Lebesgue measure. Then, we focus on the character- ization of the support of singular invariant measures. In both cases, our results apply for discrete-time and continuous-time polynomial systems. Each problem is handled through an adequate reformulation into a linear optimization problem over probability measures. We show how to solve in practice this infinite linear problem with moment relaxations. This essentially boils down to solving a hierarchy of finite-dimensional semidefinite problems. This general methodology is deeply inspired from previous research efforts. The idea of formu- lation relying on linear optimization over probability measures appears in [1], with a hierarchy of semidefinite programs also called moment-sum-of-squares or sometimes Lasserre hierarchy, whose optimal values converge from below to the infimum of a multivariate polynomial. Previous work by the second author [2] shows how to use the Lasserre hierarchy to characterize invariant measures for one-dimensional discrete polynomial systems. We extend significantly this work in the sense that we now provide a characterization on multi-dimensional semialgebraic sets, in both discrete and continuous settings. We also present some application examples together with numerical results for several dynamical systems admitting either absolutely continuous or singular invariant measures.

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Solving nearly sparse polynomial optimization problems

Minisymposium: New Hierarchies of SDP Relaxations for Polynomial Systems

T. Weisser[1] , J. B. Lasserre[2]

[1]LAAS-CNRS and University of Toulouse, [2]LAAS-CNRS and Instistute of Mathematics

We introduce the notion of a "nearly sparse" polynomial optimization problem which is an optimization problem that satisfies a certain sparsity pattern, except for a few constraints that violate this sparsity pattern. At the price of introducing some additional variables and constraints, we define an equivalent extended problem which satisfies a sparsity pattern induced by the original one. In doing so one may apply the sparsity-adapted hierarchies of convex relaxations already introduced in [1] and more recently in [2]. If the initial sparsity pattern satisfies the Running Intersection Property (RIP) then so does the induced one and therefore convergence of the sparsity-adapted hierarchies to the global optimum is guaranteed. This allows to solve large-scale nearly sparse non-convex polynomial optimization problems.

References

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Moment/sum-of-squares hierarchy for complex polynomial optimization

C. Josz[1], , D. K. Molzahn[2]

[1]LAAS CNRS, [2]Argonne National Laboratory

Multivariate polynomial optimization where variables and data are complex numbers is a non-deterministic polynomial-time hard problem that arises in various applications such as electric power systems, signal processing, imaging science, automatic control, and quantum mechanics. Complex numbers are typically used to model oscillatory phenomena which are omnipresent in physical systems. We propose a complex moment/sum- of-squares hierarchy of semidefinite programs to find global solutions with reduced computational burden compared with the Lasserre hierarchy for real polynomial optimization. We apply the approach to large-scale sections of the European high-voltage electricity transmission grid. Thanks to an algorithm for exploiting sparsity, instances with several thousand variables and constraints can be solved to global optimality.

MS 11. First order methods and applications

Monday, Time: 15.30-17.30, Room: L1.202, Building L

First-order scaled methods for image reconstruction in X-rays Computed Tomography

V. L. Coli[1] , E. L. Piccolomini[2] , E. Morotti[3] , L. Zanni[1]

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In the bio-medical domain, the X-rays 3D Computed Tomography (CT) imaging problems were recently involved by the challenge of reconstructing images from low sampling acquisitions. The main approaches to this problem are based on Iterative Image Reconstruction (IIR) methods that allow to effectively deal with important features of the problem, such as the huge size, the need to force some form of sparsity or other constraints on the image to be reconstructed. In order to apply IIR algorithms, the CT image reconstruction problem can be formulated as a constrained regularized minimization problem

$$\min_{x\geq 0} J(x) = J_0(x) + \lambda R(x) \tag{1}$$

where $J_0(x)$ denotes a fit-to-data function and $R(x)$ is a regularization term. This work focuses on problem (1) in the cases where $J_0(x)$ can be a least squares function or the Kullback-Leibler divergence, in order to adapt the model to the kind of noise affecting the data, and $R(x)$ is a smoothed Total Variation regularization function, widely employed in CT imaging problems [1]. A first-order method based on special scaled gradient directions and step-length selection rules is here applied to solve problem (1). In particular, the scaled gradient direction is obtained by means of diagonal matrices derived from suitable splitting of the gradient of $J(x)$. Moreover, two different strategies for the choice of the step-length are considered: an alternating Barzilai-Borwein rule and a selection strategy recently developed in [2]. The combination of these strategies lead to promising improvements in the convergence rate of the gradient method and numerical experiments on different test cases will be presented to evaluate effectiveness of the presented method for reconstructing X-rays 3D CT images from low-sampled data.

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P2GP: a proportioning-based two-phase gradient algorithm for QP problems with bounds on the variables and a linear constraint

M. Viola[1] , D. di Serafino[2] , G. Toraldo[3] , J. Barlow[4]

[1]Department of Computer Control and Management Engineering, Sapienza University of Rome, [2]Department of Mathematics and Physics, University of Campania "L. Vanvitelli", [3]Department of Mathematics and Applications "R. Caccioppoli", University of Naples Federico II, [4]Department of Computer Science and Engineering, Pennsylvania State University

We present a gradient-based method for Quadratic Programming problems with a Single Linear constraint and Bounds on the variables (SLBQPs):

$$\begin{array}{ll} \min & f(\mathbf{x}) := \frac{1}{2}\mathbf{x}^T H \mathbf{x} - \mathbf{c}^T \mathbf{x}, \\ \text{s.t.} & \mathbf{q}^T \mathbf{x} = b, \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \end{array} \tag{1}$$

where $H \in \mathbb{R}^{n\times n}$ is symmetric, $\mathbf{c}, \mathbf{q} \in \mathbb{R}^n$, $b \in \mathbb{R}$, $\mathbf{l} \in \{\mathbb{R} \cup \{-\infty\}\}^n$, and $\mathbf{u} \in \{\mathbb{R} \cup \{+\infty\}\}^n$. The design of efficient algorithms for (1) is of interest since SLBQPs arise in several applications (e.g., support vector machine training, portfolio selection, and image processing).

Like the GPCG algorithm for bound-constrained convex quadratic problems [4], the new method, named P2GP (*Proportioning-based 2-phase Gradient Projection*), alternates between two phases: an identification phase, which performs gradient projection iterations until either a candidate active set is identified or no reasonable progress is made, and an unconstrained minimization phase, which reduces the objective function in a suitable space defined by the identification phase [2]. The minimization phase can be performed by applying a method for unconstrained quadratic programming, such as the conjugate gradient method or the spectral gradient method proposed in [1]. Apart from solving a wider class of problems, P2GP differentiates from GPCG mainly for the criterion used to stop the minimization in the reduced space. This criterion is based on a comparison between a measure of optimality in the reduced space and a measure of bindingness of the variables that are active, defined by extending the concept of proportioning introduced for box-constrained problems [3].

Our computational experiments show a clear superiority of P2GP over a GPCG-like algorithm, in which the termination of the minimization phase is only based on the bindingness of the active variables. Proportioning allows to handle the minimization phase in a more clever way, leading in particular to a much smaller sensitivity to the Hessian condition number. From the theoretical point of view, a nice consequence of using the proportioning criterion is that finite convergence for strictly convex problems can be proved even in case of degeneracy at the solution.

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QP Algorithms with Rate of Convergence and Applications

O. Vlach[1] , Z. Dostál[1]

[1]VŠB Technical University of Ostrava

The algorithms that can solve some special classes of optimization problems with the optimal (i.e. asymptotically linear) complexity are powerful tools for the solution of large optimization problems. In this talk, we review our recent algorithms [2], [3] for the solution of quadratic programming (QP) and QPQC problems with separable convex inequality constraints and/or linear equality constraints. A nice feature of these algorithms is their capability to find an approximate solutions of such QP or QPQC problems with the spectrum of the Hessian matrix in a given positive interval at $O(1)$ matrix/vector multiplications.

If applied to the class of problems with the Hessian which admits the matrix/vector multiplication at the linear cost, then these algorithms are optimal. We also consider some improvements based on heuristic arguments. The performance of the basic algorithms and their modifications is demonstrated on the solution of large benchmarks arising from the variationaly consistent discretization of contact problems of elasticity [1].

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MS 12. Optimization methods for inverse problems in imaging

Tuesday, Time: 11.00-13.00, Room: L1.202, Building L

On Optimization of a Network of Minimal Paths for 3D Image Segmentation

J. Spencer[1] , K. Chen[1]

[1]Liverpool Centre for Mathematics in Healthcare, University of Liverpool

The accuracy of anatomical contouring of organs at risk for radiotherapy planning is of great importance, and here we consider an approach intended to reliably compute

3D segmentations in medical imaging based on a particular type of user input. Specifically, we incorporate known 2D contours from three orthogonal views. The motivation behind this approach is that acquiring an accurate contour in 2D is significantly more straightforward than acquiring an accurate surface in 3D, and so we utilise partial knowledge of the surface of the object in this way.

Given user input of this form the problem can be seen as a series of minimal surface problems, akin to the work of Grady [1]. What separates this work from [1] is the presence of orthogonal contours that essentially form an example minimal path between two points on the surface of the object. With that in mind we consider the work of Ardon and Cohen [2] that aims to construct a network of minimal paths that approximate a minimal surface, originally based on the 2D globally minimal path work of Cohen and Kimmel [3]. Here we discuss some drawbacks of the network approach for 3D image segmentation.

Our work concerns how to sequentially constrain and compute globally minimal paths such that they approximate a minimal surface. The approach we take primarily involves adjusting the weighting function in the Eikonal equation, which we solve with the fast sweeping algorithm of Zhao [4]. Here we focus on how to constrain the minimal path equations in an optimal way, such that an accurate segmentation can be computed in 3D. We also consider how to minimise the computation time, which is essential for the main application of interest.

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Automatic adaptive multi-penalty regularization for linear inverse problems

G. Landi[1] , F. Zama[1] , V. Bortolotti[2]

[1] Department of Mathematics, University of Bologna, [2] Department of Civil, Chemical, Environmental, and Materials Engineering, University of Bologna

In recent years, there has been a growing interest in multi-penalty regularization for the solution of linear inverse and ill-posed problems. The interest in multi-penalty models is due to their potential to reconstruct several distinct features of the solution. In this work, we consider two penalty terms respectively based on the ℓ_2 and ℓ_1 norm of the solution. Moreover, for further improving the solution quality, we introduce locally adapted parameters for the ℓ_2 norm-based term. Given the problem $Ax = b$ where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$, $m \geq n$, we consider a minimization problem of the form

$$\min_x \; \frac{1}{2}\|Ax - b\|_2^2 + \frac{(1-\theta)}{2}\sum_{i=1}^{n} \lambda_i (Lf)_i^2 + \theta\mu\|x\|_1 \qquad (1)$$

where λ_i, $i = 1, \ldots, n$ and μ are the regularization parameters, $L \in \mathbb{R}^{n \times n}$ is the discrete Laplacian operator and $\theta \in [0, 1]$.

In this work, we propose an algorithm to iteratively update both the regularization parameters and the approximate solution of (1). More precisely, at each iteration, the algorithm computes the regularization parameters by balancing the fidelity and penalty terms and by using the Uniform Penalty principle [1] stating that all the terms $\lambda_i(Lf)_i^2$ should be constant. An approximate solution is then obtained by solving the regularization problem (1) with the fast iterative shrinkage-thresholding algorithm (FISTA) [2]. The results of several numerical experiments are reported in order to illustrate the potential of the proposed multi-penalty approach.

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A Variational Bayesian Approach for Image Restoration with Poisson-Gaussian Noise

J.-C. Pesquet[1] , Y. Marnissi[2] , Y. Zheng[3] , E. Chouzenoux[2]

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In this talk, a methodology is investigated for signal recovery in the presence of mixed Poisson-Gaussian noise and more detailed can be found in [1].

Existing strategies for solving inverse problems often define the estimate as a minimizer of an appropriate cost function. Several algorithms have been proposed to tackle the problem of restoration for signals corrupted with non-Gaussian noise by using minimization approaches (see [2] and the references therein). In these approaches, the regularization parameter allows a tradeoff to be performed between fidelity to the observations and the prior information. The problem of selecting the regularization parameter remains an open issue especially in situations where the images are acquired under poor conditions i.e. when the noise level is very high. To address the shortcomings of these methods, one can adopt the Bayesian framework. In particular, Bayesian estimation methods based on Markov Chain Monte Carlo (MCMC) sampling algorithms have been recently extended to inverse problems involving non-Gaussian noise. However, despite good estimation performance that has been obtained, such methods remain computationally expensive for large scale problems. Another alternative approach which is explored here is to rely on variational Bayesian approximation (VBA). Instead of simulating from the true posterior distribution, VBA approaches aim at approximating the intractable true posterior distribution with a tractable one from which the posterior mean can be easily computed. These methods can lead generally to a relatively low computational complexity when compared with sampling based algorithms.

In this work, we propose such a VBA estimation approach for signals degraded by an arbitrary linear operator and corrupted with non-Gaussian noise. One of the main advantages of the proposed method is that it allows us to jointly estimate the original signal and the required regularization parameter from the observed data by providing good approximations of the Minimum Mean Square Estimator (MMSE) for the problem of interest. While using VBA, the main difficulty arising in the non-Gaussian case is that the involved likelihood and the prior density may have a complicated form and are not necessarily conjugate. To address this problem, a majorization technique is adopted providing a tractable VBA solution for non-conjugate distributions. Our approach allows us to employ a wide class of a priori distributions accounting for the possible sparsity of the target signal after some appropriate linear transformation. Moreover, it can be easily applied to several non Gaussian likelihoods that have been widely used. In particular, experiments in the case of images corrupted by Poisson Gaussian noise showcase the good performance of our approach compared with methods using the discrepancy principle for estimating the regularization parameter. Moreover, we propose variants of our method leading to a significant reduction of the computational cost while maintaining a satisfactory restoration quality.

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A block-coordinate variable metric line-search based proximal-gradient method for nonconvex optimization

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Nonconvex nondifferentiable optimization problems whose objective function is the sum of a differentiable data-fidelity term and a convex penalty term have become ubiquitous in scientific applications arising from signal and image processing. In this talk, we address the special case in which the convex part has a decomposable structure, leading to the following problem:

$$\arg\min_{x\in\mathbb{R}^n} f(x) \equiv f_0(x) + \sum_{i=1}^{m} f_i(x_i) \tag{1}$$

where $x = (x_1, \dots, x_m)$, $x_i \in \mathbb{R}^{n_i}$ with $\sum_{i=1}^{m} n_i = n$, f_0 is a continuously differentiable (possibly nonconvex) function and, for $i = 1, \dots, m$, f_i is a convex (possibly nondifferentiable) term. Problem (1) is often encountered in applications such as blind deconvolution and nonnegative matrix factorization. The standard method to tackle problem (1) is the so-called *block coordinate descent method* (also known as *nonlinear Gauss-Seidel method*), which cyclically performs the minimization of the objective function f over each block of variables x_i. In particular, at each iteration $k \in \mathbb{N}$, the iterate $x^{(k+1)} = (x_1^{(k+1)}, \dots, x_m^{(k+1)})$ is computed such that each component $x_i^{(k+1)}$,

$i = 1, \ldots, m$, is a solution of the subproblem

$$x_i^{(k+1)} \in \arg\min_{x \in \mathbb{R}^{n_i}} f_0(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, x, x_{i+1}^{(k)}, \ldots, x_m^{(k)}) + f_i(x). \qquad (2)$$

The applicability of scheme (2) is quite limited, due to the fact that computing the exact minimum of f, even if restricted to a single block of variables, may be impracticable in most cases. To overcome this limitation, subproblem (2) can be replaced by a proximal-gradient step, giving rise to the *block coordinate proximal-gradient method* [1,2]. This second approach is attractive due to its simplicity and low computational cost per iteration, however it usually suffers from slow convergence and, in addition, requires a closed formula for the computation of the proximity operator of f_i, which is often unavailable.
In this talk, we propose a novel version of the block-coordinate descent method [3] where subproblem (2) is solved inexactly by means of a fixed number of steps of the variable metric proximal-gradient method proposed in [4], whose general iteration for the $i-$th block is given by

$$x_i^{(k,\ell+1)} = x_i^{(k,\ell)} + \lambda_i^{(k,\ell)}(u_i^{(k,\ell)} - x_i^{(k,\ell)}), \quad \ell = 0, \ldots, L_{i-1}$$

where $u_i^{(k,\ell)}$ is a suitable approximation of the proximal-gradient step, i.e.

$$u_i^{(k,\ell)} \approx \mathrm{prox}_{\alpha_i^{(k,\ell)} f_i}^{D_i^{(k,\ell)}} \left(x_i^{(k,\ell)} - \alpha_i^{(k,\ell)} (D_i^{(k,\ell)})^{-1} * \right.$$
$$\left. \nabla f_0 \left(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, x_i^{(k,\ell)}, x_{i+1}^{(k)}, \ldots, x_m^{(k)} \right) \right)$$

being $\alpha_i^{(k,\ell)}$ a positive steplength parameter and $D_i^{(k,\ell)}$ a symmetric positive definite matrix, while $\lambda_i^{(k,\ell)}$ is computed via a certain Armijo-like condition. The novelty of our approach is that the parameters $\alpha_i^{(k,\ell)}$ and $D_i^{(k,\ell)}$ can be computed with a relative freedom of choice, without relating them to the Lipschitz constant or using a priori formulas, in order to accelerate the convergence rate of the entire scheme.
We will show that the newly proposed method is both mathematically sound and computationally efficient. From the theoretical point of view, we prove the global convergence of the algorithm by assuming that the proximal-gradient points are computed more and more accurately as the iterations go. Furthermore, like other similar approaches in the literature [5], the strong convergence of the algorithm to a critical point holds whenever the objective function satisfies the Kurdyka–Łojasiewicz inequality at its limit points and the gradient of the differentiable part is assumed to be locally Lipschitz continuous. Finally, from the practical viewpoint, we provide some numerical results in image processing applications where our method effectively accelerates the progress towards the solution of the problem.

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MS 13. Geometric control and applications

Monday, Time: 15.30-17.30, Room: L1.201, Building L

Bang–bang–singular extremals in Mayer problems

L. Poggiolini[1] , G. Stefani[1]

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In this talk we consider a Mayer problem on a fixed time interval for a control affine dynamics on a smooth manifold M. We model the system as follows: let $X_1, \dots, X_m$ be smooth vector fields on M and let $\mathcal{X}$ be their convex hull, i.e.

$$\mathcal{X}(x) = \Big\{ \sum_{i=1}^{m} u_i X_i(x) \colon u = (u_1, \dots, u_m) \in \Delta \Big\},$$

where $\Delta := \big\{ u = (u_1, \dots, u_m) \in \mathbb{R}^m \colon u_i \geq 0,\ i = 1, \dots, m,\ \sum_{i=1}^m u_i = 1 \big\}$ is the unit symplex of $\mathbb{R}^m$. Given a smooth function $c \colon M \to \mathbb{R}$, a point $x_0 \in M$ and a manifold $N_f \subset M$, we consider a Mayer problem of the following kind

$$\text{minimize} \quad c(\xi(T)) \text{ subject to} \tag{1a}$$

$$\dot{\xi}(t) \in \mathcal{X}(\xi(t)) \quad \text{a.}e.\ t \in [0, T], \tag{1b}$$

$$\xi(0) = x_0, \quad \xi(T) \in N_f. \tag{1c}$$

For such problem we assume we are given a bang-bang-partially singular reference trajectory satisfying the necessary conditions for optimality and we give sufficient conditions for its strong local optimality.

Our sufficient conditions tantamount to regularity conditions - both on the arcs and at the switching points between two consecutive arcs - and to the coercivity of an appropriate second variation associated to a subproblem of the given one. The sufficiency of such conditions is proven via Hamiltonian methods.

Π-Singularities in Minimum Time Control applied to Space Mechanic.

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In this talk, we will adress the issue of π-singularities, a phenomenon occuring when one is dealing with minimum time control of a mechanical system. More detailed can be found in [1, 2].

Minimum time control of planar two and three body problems comes down to tackle the following affine controlled system :

$$\begin{cases} \dot{x} = F_0(x) + u_1 F_1(x) + u_2 F_2(x), x \in M, \|u\| \leq 1, \\ x(0) = x_0, \\ x(t_f) = x_f, \\ t_f \to \min. \end{cases} \tag{1}$$

Where M is the phase space (4 dimensional), $u = (u_1, u_2)$ is the control, F_0 is the drift coming from the gravitational potential, and F_1, F_2 the two orthogonal vector fields supporting the control. This implies convenient properties on the vector fields and their Lie brackets :

$(A_1) : \forall x \in M, \mathrm{rank}(F_1(x), F_2(x), [F_0, F_1](x), [F_0, F_2](x)) = 4,$

Necessary conditions coming from the Pontryagin Maximum Principle leads to study the flow of a singular Hamiltonian problem given by

$$H(x,p) = H_0(x,p) + \sqrt{H_1^2(x,p) + H_2^2(x,p)},$$

with $H_i(x,p) = \langle p, F_i(x) \rangle, i = 0, 1, 2$, being the Hamiltonian lift of the vector fields. It also provides the control feedback $u = \frac{(H_1, H_2)}{\|(H_1, H_2)\|}$ outside of the singular locus $\Sigma = \{H_1 = H_2 = 0\}$.

For extremal crossing Σ, three cases are to be distinguish :

- around $\bar{z} \in \Sigma$ such that $H_{12}(\bar{z})^2 < H_{01}^2(\bar{z}) + H_{02}^2(\bar{z})$, the extremal flow can be stratified, and a co-dimension one sub-manifold is leading to the singular locus. The flow is smooth on each strata and continuous.
- around $\bar{z} \in \Sigma$ such that $H_{12}(\bar{z})^2 > H_{01}^2(\bar{z}) + H_{02}^2(\bar{z})$ no extremal is crossing the singular locus, and the flow is smooth.
- in the rare limit case where $H_{12}(\bar{z})^2 = H_{01}^2(\bar{z}) + H_{02}^2(\bar{z})$ no extremal is crossing the singular locus either.

The controlled restricted three body problem has a simpler structure since we can add hypothesis (A_2) : $[F_1, F_2] = 0$ identically, as such, we find ourselves in the first case.

From the feedback given above and the involution condition (A_2), it appears that when Σ is crossed, we get an instant rotation of angle π on the control, and the flow is stratified as above. This stratification will also answer the question of which trajectories leads to a singularity.

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Averaging for minimum time control problems and applications

J. Rouot[1] , J.-B. Caillau[2] , J.-B. Pomet[3]
[1]LAAS-CNRS Toulouse, [2]Univ. Bourgogne Franche Comt & INRIA, [3]INRIA Sophia Antipolis

The work is motivated by the study of the minimum time orbital transfer problem in geometric optimal control theory. We consider a system of slow/fast variables, typically, the elliptic motion of a satellite in a central field: an angle which locates the satellite on its orbit has fast time evolution with respect to the elements characterizing the geometry of the orbit. On top of that, we add some perturbations coming from a control function which appear linearly in the dynamics and a drift term on the slow and fast variables. The Pontryagin Maximum Principle (PMP) provides an extremal flow governing the evolution of minimizing trajectories. In order to facilitate the study of the associated system, we apply classical techniques of averaging and define an average system of the extremal system given by the PMP (see also [2]). It amounts to study a boundary value problem which can be treated numerically and be used to find extremals of the non average system. Also, we give some convergence results to relate the non average extremal system with the average system. We analyze some metric properties of the average flow, showing that with no drift on the slow variables, it defines a (symmetric) Finsler metric on the set of extremals (see [1]). Then, adding a drift term, we study the deformation of this metric and present numerical results concerning the value function of the non average system.

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MS 14. Non-smooth optimization and computational 3D image

Thursday, Time: 8.00-10.00, Room: L1.201, Building L

Block-proximal methods with spatially adapted acceleration

T. Valkonen[1]
[1]University of Liverpool

We study a class of primal–dual optimisation algorithms with spatially adapted step length parameters and acceleration: for example, in deblurring problems, we can accelerate each Fourier component of the solution based on the corresponding weight of the blurring filter. In problems whose primal–dual formulations involve additional slack variables, such as TGV^2 reconstruction problems, we can separately accelerate the interesting original variable, and the slack variables. Our methods can be executed either stochastically or deterministically. Even without full strong convexity, we observe mixed $O(1/N^2) + O(1/N)$ performance. We demonstrate the proposed methods on various image processing problems.

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Accelerated and Preconditioned Douglas–Rachford Algorithms for the Solution of Variational Imaging Problems

K. Bredies[1] , H. Sun[2]

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We discuss basic, accelerated and preconditioned versions of the Douglas-Rachford (DR) splitting method for the solution of convex-concave saddle-point problems that arise in variational imaging. While the basic DR iteration admits weak and ergodic convergence with rate $O(1/k)$ for restricted primal-dual gaps, acceleration leads to convergence rates of $O(1/k^2)$ and $O(q^k)$ for some $0 < q < 1$ under appropriate strong convexity assumptions. Further, preconditioning allows to replace the potentially computationally expensive solution of a linear system in each iteration step in the corresponding DR iteration by fast approximate solvers without the need to control the error.

The methods are applied to non-smooth and convex variational imaging problems. We discuss denoising and deconvolution with L^2 and L^1 discrepancy and total variation (TV) as well as total generalized variation (TGV) penalty. Preconditioners which are specific to these problems are presented, the results of numerical experiments are shown and the benefits of the respective accelerated and preconditioned iteration schemes are discussed.

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Total Variation Image Reconstruction on Smooth Surfaces

M. Herrmann[1] , R. Herzog[2] , H. Kröner[3] , S. Schmidt[1] , J. Vidal[2]

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An analog of the total variation image reconstruction approach [3] for images f, defined on smooth surfaces, is introduced, which can be used as a application for 3D scanner data of objects and their textures. A field, where the texture is of great importance, is neuroimaging, where random noise enters due to eddy-current distortions or physical motion during the magnetic resonance imaging (MRI) process. For this purpose, we consider the image reconstruction problem

$$\left\{ \begin{array}{rl} \text{Minimize} & \dfrac{1}{2}\displaystyle\int_S |Ku - f|^2 \,\mathrm{d}s + \dfrac{\alpha}{2}\int_S |u|^2 \,\mathrm{d}s + \beta \int_S |\nabla u| \\ \text{over} & u \in BV(S), \end{array} \right. \tag{1}$$

where $S \subset \mathbb{R}^3$ is a smooth, compact, orientable and connected surface without boundary. The observed data $f \in L^2(S)$, parameters $\beta > 0$, $\alpha \geq 0$ and the observation operator $K \in \mathcal{L}(L^2(S))$ are given. Furthermore, $BV(S)$ denotes the space of functions of bounded variation on the surface S. A function $u \in L^1(S)$ belongs to $BV(S)$ if the TV-seminorm defined by

$$\int_S |\nabla u| = \sup\left\{ \int_S u \operatorname{div} \boldsymbol{\eta} \,\mathrm{d}s : \boldsymbol{\eta} \in \boldsymbol{C}_c^\infty(\operatorname{int} S, TS),\ |\boldsymbol{\eta}(p)|_2 \leq 1 \ \forall p \in S \right\}$$

is finite, where TS is the *tangent bundle* of S. Note that $BV(S) \hookrightarrow L^2(S)$ and hence, the integrals in (1) are well defined. The non-smoothness of TV-seminorm is dealt with a duality approach, see [1, 2]. This leads to the predual problem of (1), which is a quadratic optimization problem for the vector field $\boldsymbol{p} \in \boldsymbol{H}(\operatorname{div}; S) := \{\boldsymbol{v} \in \boldsymbol{L}^2(S; TS) \mid \operatorname{div} \boldsymbol{v} \in L^2(S)\}$ with pointwise inequality constraints on the surface.

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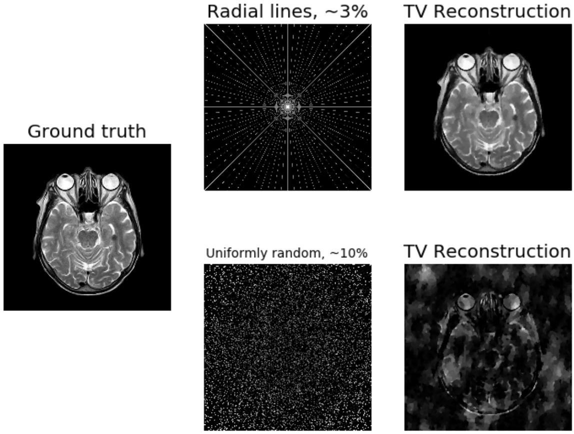
Learning the Sampling Pattern for MRI

F. Sherry[1] , M. J. Ehrhardt[1] , M. Benning[1] , G. Maierhofer[1] , C. B. Schönlieb[1]

[1] University of Cambridge

Taking measurements in MRI is a time-consuming procedure, so ideally one would take few samples and still recover a useable image. It is crucial that these samples are positioned in frequency space in a way that allows as much information to be extracted from the samples as possible. The choice of what frequencies to sample can have a large influence on the reconstruction quality (as is shown in the figure below).



We consider the problem of determining a suitable sampling pattern for a class of objects that are in some sense similar. Given a training set (consisting of pairs of clean images and corresponding MRI measurements) that is representative of the class of objects that we are trying to image, a bilevel optimisation problem can be formulated, the objective of which is to learn a sparse sampling pattern that works well for this class. In this bilevel problem the upper problem measures the reconstruction quality (on the training set) and penalises the lack of sparsity of the sampling pattern and the lower problem is the variational problem that is solved to reconstruct images from MRI measurements (such as a total variation regularised least squares problem). The sampling pattern learned by solving the bilevel problem can be used to accelerate imaging of objects similar to those in the training set by reducing the number of measurements that need to be taken.

8. Contributed Talks

CT 1. Vector-value optimization and related issues

Tuesday, Time: 14.00-16.00, Room: L1.202, Building L

Local-global type properties for generalized convex vector functions

N. Popovici[1] , O. Bagdasar[2]

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Generalized convexity plays an important role in vector optimization, equilibrium problems and other related topics. Among many classes of generalized convex functions known in the literature, the (semistrictly/explicitly) quasiconvex functions are of special interest, as they preserve some fundamental properties of convex functions:

• every semistrictly quasiconvex real-valued function satisfies the so-called "*local min - global min*" property, i.e., its local minimizers are actually global minimizers (see, e.g., Ponstein [5]);

• every explicitly quasiconvex real-valued function satisfies the so-called "*local max - global min*" property, namely any local maximizer is a global minimizer, whenever it belongs to the intrinsic core of the function's domain (see, e.g., Bagdasar and Popovici [2]).

In this talk we present generalizations of these two extremal properties for vector functions with respect to the concepts of ideal, strong and weak optimality, currently used in multiobjective optimization. We show that the "*local min - global min*" property can be actually extended to different classes of semistrictly quasiconvex vector-valued functions, while the "*local max - global min*" property can be generalized for componentwise explicitly quasiconvex vector functions.

As applications of these results we obtain new insights on the structure of ideal, strong and weakly optimal solution sets in multicriteria linear fractional programming.

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Robust counterparts of the Markowitz portfolio optimization problem via utility functions

E. Moretto[1] , N. Popovici[2]

[1]Department of Economics, University of Insubria, [2]Department of Mathematics, Babeş-Bolyai University

The optimal portfolio problem, as formulated by Markowitz [1], plays an important role in financial mathematics. Being intimately related to bicriteria optimization, it can be approached following recent trends in multiobjective optimization theory concerning robustness with respect to certain uncertainty parameters.

In this setting, stocks are denoted by means of their their random one-period rate of return $\tilde{r}_i$, $i = 1, ..., n$ and represented by their first- and second-order moments, that is expected rates of return $\bar{r}_i$, variances σ_i^2 and covariances $\sigma_{i,j}$, $i, j = 1, ..., n$. A portfolio P is a combination of stocks, i.e, a vector $\mathbf{x} \in \mathbb{R}^n$ whose random return is $\tilde{r}_P = \sum_{j=1}^{n} \tilde{r}_j x_j$. The expected return of P is a linear form while its variance is, instead, a quadratic form.

A rational agent wishes to determine the 'best' portfolio. This is achieved in two ways:

- firstly, the set of 'efficient' portfolios is to be found; these portfolios are not dominated in terms of mean-variance (a portfolio P dominates a portfolio Q if $\bar{r}_P > \bar{r}_Q$ and, at the same time, $\sigma_P^2 < \sigma_Q^2$).
- secondly, the preferred portfolio is the one that best matches the risk attitude of the agent.

This choice can be done applying a quadratic utility function that is increasing (decreasing) with respect to $\bar{r}_P$ (σ_P^2).

A number of recent papers (see, for instance, [2]) analyze the optimal portfolio problem by exploiting recent results in robust multiobjective optimization by means of the weighted-sum or the ε-constraint scalarization methods. To the best of our knowledge, this stream of literature has, so far, focused mainly on the first part of the optimal portfolio search.

The aim of this talk is to introduce new robust counterparts of the optimal portfolio problem by means of certain utility functions instead of the weighted-sum or the ε-constraint scalarization methods. Our approach gives new insights on the risk attitude of the agent.

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A Forward-Backward Method for Solving Vector Optimization Problems

S.-M. Grad[1]

[1]Chemnitz University of Technology

We present an iterative proximal inertial forward-backward method with memory effects, based on recent advances in solving scalar convex optimization problems and monotone inclusions, for determining weakly efficient solutions to convex vector optimization problems consisting in vector-minimizing the sum of a differentiable vector function with a nonsmooth one, by making use of some adaptive scalarization techniques.

Jahn-Graef-Younes type algorithms for discrete vector optimization based on cone-monotone sorting functions

C. Günther[1] , N. Popovici[2]

[1]Martin Luther University Halle-Wittenberg, Faculty of Natural Sciences II, Institute for Mathematics, [2]Babeş-Bolyai University, Faculty of Mathematics and Computer Science

In this talk we present new Jahn-Graef-Younes type algorithms for solving discrete vector optimization problems. In order to determine the minimal elements of a finite set with respect to an ordering cone, the original algorithmic approach by Jahn [2, 4, 3] consists of a forward iteration (Graef-Younes method [5, 3]) in a first phase, followed by a backward iteration in a second one. Our approach is based on a pre-sorting scheme via certain cone-monotonic functions. We derive new implementable algorithms for solving discrete vector optimization problems. In particular, we analyze the case where the ordering cone is polyhedral. In our computational studies we use the "Multiobjective Search Algorithm with Subdivision Technique (MOSAST)" (see Jahn [2]) that is based on the subdivision technique introduced by Dellnitz et al. in [1]. Using MOSAST it is possible to approximate the set of Pareto efficient solutions of continuous multiobjective optimization problems by applying algorithms for discrete vector optimization problems. We present computational results obtained by our algorithms in MATLAB and make comparisons with the original Jahn-Graef-Younes method.

References

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Copositive Optimization in Infinite Dimension

C. Adams[1] , M. Dür[1] , L. Frerick[1]
[1]Department of Mathematics, University of Trier

This talk will address a duality theory for copositve optimization problems on infinite graphs and its analytic background. More details can be found in [1, 2].

It is well known, that combinatorial optimization problems on finite graphs can be formulated as conic convex optimization problems (e.g. stable set, maximum clique, maximum cut). Especially NP-hard problems can be written as copositive programs. In this case the hardness is moved into the copositivity constraint.
Copositive programming is a quite new topic in optimization. It deals with optimization over the so-called copositive cone, a generalization of the positive semidefinite cone:

$$\mathcal{COP}_n := \left\{ A \in \mathbb{R}^{n \times n} \colon A = A^\top,\, x^\top A x \geq 0 \text{ for all } x \in \mathbb{R}^n_+ \right\}.$$

Its dual cone is the completely positive cone:

$$\mathcal{CP}_n := \left\{ \sum_{i=1}^{k} d_i d_i^\top : d_i \in \mathbb{R}^n_+ \right\}.$$

The related copositive optimization problem is a linear program with matrix variables and has the following form:

$$\begin{aligned} &\min \langle C, X \rangle \\ &\text{s.t. } \langle A_i, X \rangle = b_i \ (i = 1, \ldots, m), \\ &\qquad X \in \mathcal{COP}_n. \end{aligned}$$

However, some optimization problems can be formulated as combinatorial problems on infinite graphs. An example is the kissing number problem, which can be formulated as a stable set problem on a circle. We will show how to lift the theory of copositive optimization to infinite dimension and we study operators in Hilbert spaces instead of matrices. This new theory will use tools from functional analysis, but the restriction to finite dimension provides the usual duality theory.
One approach is introduced in [2]. Here the generalization of the copositive cone $\mathcal{COP}_n$ is defined as the cone of copositive kernels $\mathcal{COP}_V$. Let V be a compact metric space with Borel measure ω, then

$$\mathcal{COP}_V := \left\{ K \in C(V \times V)_{\text{sym}} \colon \int_V \int_V K(x,y) f(x) f(y) d\omega(x) d\omega(y) \geq 0 \ \forall f \in C(V)_{\geq} \right\}$$

Its dual cone is the cone of completely positive measures $\mathcal{CP}_V$:

$$\mathcal{CP}_V = \text{cl} \left\{ \sum_{\nu=1}^{N} \mu_\nu \otimes \mu_\nu \colon \mu_\nu \in M(V),\, \mu_\nu \geq 0 \right\}.$$

In this context we discuss some properties which are equivalent to the ones in finite dimension and we also point out differences. We will show some special cases for which the theory can be formulated very well and some cases for which it is not easy. Understanding these cases and their special properties is important for developing solution approaches for problems of that kind.

We will discuss an essential and popular norm in functional analysis, the symmetric projective norm, and its relevant properties. Furthermore we will explain its importance for our duality theory.

The projective norm π_p^s on the symmetric tensor space $(\mathbb{R}^n \otimes_{\mathrm{s}} \mathbb{R}^n) = \mathbb{R}^{n\times n}_{\mathrm{sym}}$ is given by:

$$\pi_p^s\,(z) := \inf\left\{\sum_{\nu=1}^{N} \|x^{(\nu)}\|_p^2 \colon N \in \mathbb{N}, z = \sum_{\nu=1}^{N} \pm\, x^{(\nu)} \otimes x^{(\nu)},\, x^{(\nu)} \in \mathbb{R}^n\right\}.$$

In [2] the special case $p = 1$ is discussed. We will embed this case in a general theory and explain why it is particular. Furthermore we will show another interesting case, $p = 2$, which will be discussed in [1]. In both cases nice properties are given and a relation to the completely positive cone is apparent.

References

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Multiobjective optimization with uncertainties and the application to reduced order models for PDEs

S. Peitz[1] , D. Beermann[2] , M. Dellnitz[1] , S. Volkwein[2]

[1]Universität Paderborn, [2]Universität Konstanz

We present a gradient-based algorithm for the solution of multiobjective optimization problems with uncertainties. To this end, an additional condition is derived for the descent direction in order to account for inaccuracies in the gradients and then incorporated in a subdivison algorithm for the computation of global solutions to multiobjective optimization problems. Convergence to a superset of the Pareto set is proved and an upper bound for the maximal distance to the set of substationary points is given. Finally, the results are coupled with error estimates for POD-based reduced order models for PDEs in order to efficiently solve PDE-constrained multiobjective optimal control problems.

CT 2. PDE-based optimization

Tuesday, Time: 14.00-16.00, Room: Hall L1, Building L

Optimal Control of Elastoplastitcity Problems with Finite Deformations and Application to Deep Drawing Processes

A. Walter[1] , S. Ulbrich[1]

[1]TU Darmstadt

We consider the optimal control of deep drawing processes in the context of sheet metal forming. This process is modeled as an elastoplasticity problem with frictional contact.

Due to the high drawing depth, common approaches for infinitesimal deformations are not sufficient, thus we deal with finite deformations. For this purpose we use a hyperelastic material model where the deformation gradient is multiplicative decomposed in an elastic and a plastic part. The resulting model can be written as a quasivariational inequality of mixed kind. For a numerical treatment we use an implicit time discretization scheme and a semi-smooth reformulation of the quasivariational inequality. We solve this formulation by applying a semi-smooth Newton method in each time step. Our aim is to optimize the deep drawing process such that we get an optimal die filling while avoiding failure modes of the forming process like cracking or leaking of the tool system. We are able to control the process via the internal pressure and the blank holder force. This results in an optimal control problem which can be solved with a bundle trust region method. For this we use an adjoint based subgradient computation. Furthermore, we introduce reduced order models for finite strain plasticity and nonlinear elasticity with contact to speed up the simulation and thereby the optimization.

References

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Space-time discretization of a parabolic optimal control problem with state constraints

C. Löbhard[1] , S. Hajian[2] , M. Hintermüller[2]

[1]Weierstrass Institute for Applied Analysis and Stochastics,
[2]Humboldt-Universität zu Berlin

We present a space-time discretization which is based on a reformulation of the stationarity conditions of a Moreau-Yosida regularized parabolic optimal control problem, which involves only the state variable. The resulting nonlinear partial differential equation is of fourth order in space, and of second order in time. In order to cope with the disbalance of regularity of the respective solutions, we develop a taylored discontinuous Galerkin scheme and derive convergence rates in the mesh size as well as an integrated update strategy for the regularization parameter related to the state constraints. We also propose an adaptive mesh refinement strategy and illustrate the performance of our method in numerical test cases.

Backward Step Control globalization for Newton-type methods

A. Potschka[1]

[1]Interdisciplinary Center for Scientific Computing, Heidelberg University

We present a step size selection method [2] for the globalization of convergence of Newton-type methods for nonlinear root finding problems and its generalization to a Hilbert space setting. The approach generalizes on the well-known interpretation of the Newton method as an explicit Euler time-stepping scheme for the Davidenko differential equation [1] to the setting of Newton-type directions, which may deviate from the exact Newton direction. The advantages of the globalization strategy comprise a simple yet efficient implementation and a rigorous convergence analysis based on generalized Newton paths, which are the solution to an inital value problem of an appropriate generalization of the Davidenko differential equation. Under reasonable assumptions, the results of the convergence analysis include

- full steps in the vicinity of a solution,
- a uniform lower step size bound,
- a priori guarantees for the nonlinear decrease of the residual norm,
- and convergence to a distinguished root, namely the end point of the generalized Newton path emanating from the initial guess.

In addition, the framework delivers suitable relative termination tolerances for inexact residual minimizing Krylov–Newton methods and can be utilized to derive general purpose adaptive grid refinement strategies for finite element discretizations of elliptic PDE problems. We conclude the talk with numerical results for the unconstrained optimization problems of the CUTEst test set and for the minimal surface PDE.

References

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Automatic Insertion of Casing Holes in TU Berlin TurboLab Stator Blade with Differentiated Open CASCADE Technology CAD System

M. Banovic[1] , S. Auriemma[2] , A. Walther[1] , J.D. Müller[3]

[1]Universität Paderborn, [2]Open CASCADE, [3]Queen Mary University of London

The TU Berlin TurboLab Stator optimisation test-case is characterised by several manufacturing constraints to be respected [1]. One of them requires that the thickness of the blade has to be defined such that two fixture holes (cylinders) can be inserted into the optimised blade. It is very complex to respect this constraint during the blade optimisation. A possible solution is to insert the holes a posteriori. To achieve this goal, one can perform an optimisation that finds the best position of the holes in the optimised blade. At the beginning of this optimisation, the holes intersect the blade and the total intersecting area is evaluated. This area is minimised by changing the holes position during the optimisation. Once the target is achieved, one can verify whether the intersections between the holes and the blade are still occurring. If there are still some intersections, one can increase the blade thickness in correspondence to the intersected areas. This would cause a slight modification of the blade geometry. For this fitting problem, we are using gradient information provided by the differentiated Open CASCADE Technology CAD system [2], in which the blade geometry is also parametrised.

References

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A Local Discontinuous Galerkin Method for Dirichlet Boundary Control Problems

H. Yücel[1]

[1]Institute of Applied Mathematics, Middle East Technical University

In this talk, we consider Dirichlet boundary control of a convection–diffusion equation with L^2–boundary controls subject to pointwise bounds on the control posed on a two dimensional convex polygonal domain. We use local discontinuous Galerkin method as a discretization method. We derive a priori error estimates for the approximation of the Dirichlet boundary control problem on a polygonal domain. Several numerical results are provided to illustrate the theoretical results.

Increasing Accuracy in Material Parameter Estimation with Algorithmic Differentiation

B. Jurgelucks[1]

[1]Mathematics and its Applications, Institute for Mathematics, Paderborn University

One obstacle in the development of new piezoelectric devices is that a cheap, easy and accurate method to identify all material parameters of piezoelectric ceramics is not readily available. A promising idea is to estimate the parameters by solving an inverse problem. However, this inverse problem cannot yet be solved to full extent.

In order to improve the situation two main optimization problems are considered for this talk: Firstly, the solution process of the inverse problem is aided directly by providing accurate derivatives via algorithmic differentiation. Secondly, the solution process of the inverse problem is aided indirectly [1, 2, 3] by solving a separate optimization problem: Increasing the sensitivity of the measurable quantity of the inverse problem with respect to all material parameters with the geometry of the piezoelectric specimen as variables to the optimization problem. In this talk we will show recent advances and results for the identification of piezoelectric material parameters.

References

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inversen Materialparameterbestimmung für Piezokeramiken. *tm-Technisches Messen*, 82(2): 102–109, 2015.

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CT 3. Optimization algorithms I

Tuesday, Time: 14.00-16.00, Room: L1.201, Building L

A Primal-Dual Augmented Lagrangian Penalty-Interior-Point Filter Line Search Algorithm

R. Kuhlmann[1] , C. Büskens[1]

[1]Zentrum für Technomathematik, Universität Bremen

Interior-point methods have been shown to be very efficient for large-scale nonlinear programming. The combination with penalty methods increases their robustness due to the regularization of the constraints caused by the penalty term. In this presentation a primal-dual penalty-interior-point algorithm is presented, that is based on an augmented Lagrangian approach with an $\ell 2$-exact penalty function. Global convergence is maintained by a combination of a merit function and a filter approach. Unlike other filter methods no separate feasibility restoration phase is required. The algorithm has been implemented within the solver WORHP to study different penalty and line search options and to compare its numerical performance to two other state-of-the-art nonlinear programming algorithms, the interior-point method IPOPT and the sequential quadratic programming method of WORHP. The proposed method is equally efficient than IPOPT on the CUTEst test set, but outperforms it on an infeasible version of it.

LDL^T Factorization of Saddle-point Matrices in Nonlinear Optimization—Reusing Pivots and Monitoring Stability

J. Kuřátko[1]

[1]Institute of Computer Science, The Czech Academy of Sciences, Faculty of Mathematics and Physics, Charles University

Iterative methods for nonlinear optimization usually solve a sequence of linear systems. This talk will address the application of direct methods in solving

$$A_k x_k = y_k, \quad k = 1, 2, \ldots,$$

where the matrices A_k are symmetric and indefinite. Moreover, consecutive matrices are related such that the structure of blocks and bands of nonzero elements remains the same. In our setup, we solve the linear system by computing the factorization $P_k A_k P_k^T = L_k D_k L_k^T$ by the Bunch-Parlett [2] or Bunch-Kaufman [1] method, where L_k is unit lower-triangular and D_k is diagonal with 1×1 and 2×2 pivots.

We will describe how to use the computed P_k in the next iteration and factorize $P_k A_{k+1} P_k^T = L_{k+1} D_{k+1} L_{k+1}^T$, where D_{k+1} features the same pattern of pivots as D_k. Such a factorization may not exist or it may be unstable. We will show one strategy for the monitoring of pivots and how to update P_k to the next iteration when needed.

In addition we will present a method that adaptively switches from the straightforward LDL^T factorization with no pivoting to Bunch-Parlett or Bunch-Kaufman. In order to speed up the computation by skipping the search for pivots we apply our monitoring strategy and if appropriate reuse the permutation from the previous iteration. A more detailed description can be found in [3]. We will conclude our talk with numerical results from a series of benchmark problems on dynamical systems optimization.

References

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Comparing solvers for unconstrained and box constrained optimization

M. Kimiaei[1] , A. Neumaier[1]

[1]Faculty of Mathematics, University of Vienna

There are a large number of publicly available solvers for unconstrained and box constrained optimization. We present a thorough comparison of many of these solvers on the CUTEst test collection of optimization problems.

References

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From least squares solutions of linear inequality systems to convex least squares problems

J.-B. Hiriart-Urruty[1]

[1]University Paul Sabatier

In our contributions, with various co-authors, we consider least squares solutions (if any) in problems of three different types: for a finite number of linear inequality systems [1], for infinitely many linear systems [2], for general convex systems [3]. Our objective focussed on existence results (for properly defined least squares solutions), characterization of such solutions, and in some cases algorithms for computing them.

References

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Algorithmic differentiation for machine learning applications in Python

K. Kulshreshtha[1] , S.H.K. Narayanan[2] , K. MacIntyre[3]

[1]Universität Paderborn, [2]Argonne National Laboratory, [3]Northwestern University

Machine learning is being employed in an increasing number of applications in the recent years. The machine learning community has adopted Python as their language of choice for rapid prototyping and testing of both applications as well as algorithms. A Stochastic Quasi Newton Method was recently proposed [4] in order to train a machine learning system with data-sets. This algorithm was shown to converge efficiently if the required derivatives could be provided efficiently. A Python implementation of the algorithm with hand coded derivatives for a certain application was also available [3]. Algorithmic Differentiation [5] is a well known technique to compute derivatives for functions represented as computer programs. The software tool ADOL-C [6] has been developed over many years to be able to provide accurate derivatives for arbitrary programs written in C and C++. The tool SWIG [1, 2] was recently developed to be able to interface the capabilities of libraries in C/C++ with applications in Python. We have used SWIG to successfully interface the capabilities provided by ADOL-C in Python to compute derivatives in the setting of the Stochastic Quasi Newton Method in order to solve machine learning problems efficiently. The details of the algorithm and tools as well as numerical results will be given in this talk.

References

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CT 4. Nonsmooth Optimization

Wednesday, Time: 13.30-15.30, Room: L1.202, Building L

Optimality Conditions for Optimization Problems with Nonsmooth Constraints

L. Hegerhorst[1]

[1]Institute of Applied Mathematics, Leibniz Universität Hannover

We consider nonsmooth optimization problems where the nonsmoothness can be formulated in terms of the absolute value function. Then the optimization problem can be recast in so-called abs-normal form. For the class of unconstrained nonlinear nonsmooth minimization there have recently been developed necessary and sufficient first and second order optimality conditions. We extend the theory to nonsmooth constrained optimization and discuss illustrative examples where this type of nonsmoothness arises in practical optimization problems.

References

[1] A. Griewank and A. Walther. First- and second-order optimality conditions for piecewise smooth objective functions. *Optimization Methods and Software*, 31(5):904–930, 2016.

DC Programming via Piecewise Linear Approximations

M. Gaudioso[1] , G. Giallombardo[1] , G. Miglionico[1] , A.M. Bagirov[2]

[1]DIMES, Università della Calabria, [2]Federation University Australia

Minimization of DC (Difference of Convex) functions is a global optimization problem of great relevance in applied areas such as classification, clustering and image processing. We focus on the numerical algorithms for finding a local minimum of a DC, not necessarily smooth, function of several variables, by introducing appropriate piecewise affine approximations of the objective function. We describe an algorithm that benefits from many similarities with the well established class of "Bundle" methods. The main characteristics of our approach is in the alternative use of two different convex models, depending on a measure of their "agreement" with the objective function. We discuss the computational results on a set of academic benchmark test problems

References

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DC Programming and DCA for Solving Quadratic Programs with Linear Complementarity Constraints

T.M.T. Nguyen[1] , H.A.L. Thi[1] , T.P. Dinh[2]

[1]Laboratory of Theoretical and Applied Computer Science (LITA), UFR MIM, University of Lorraine, [2]Laboratory of Mathematics, INSA-Rouen, University of Normandie

We investigate an important special case of the Mathematical Program with Equilibrium Constraints (MPEC), namely the Quadratic Program with Linear Complementarity Constraints (QPLCC). This problem consists in minimizing a quadratic function on a set defined by linear constraints and linear complementarity constraints. We reformulate the QPLCC via two penalty functions, one of which leads to a standard DC (Difference of Convex functions) program, the other results in a DC program with DC constraints. The DCA (DC Algorithm) schemes are developed to solve the resulting problems. Numerical experiments on several quadratic problems with linear complementarity constraints in MacMPEC, a collection of MPEC test problems in AMPL, show that the proposed DCA schemes are quite efficient in comparison with KNITRO, an advanced solver for nonlinear optimization problems including mathematical programs with complementarity constraints.

Functions of Bounded Variation on Nonsmooth Surfaces

J. Vidal[1] , R. Herzog[1] , S. Schmidt[2] , M. Herrmann[2]

[1]Technische Universität Chemnitz, [2]Universität Würzburg

In this talk we extend part of the research done in the renew paper [4] into the nonsmooth setting. Here, we allow the surface to have kinks, which requires a more demanding study based on differential geometry; see for instance [3, 5] and the references therein. In this framework, Lipschitz functions will play a deterministic role and will make it possible the definition of Lipschitz charts over a simplicial triangulation of the surface. Furthermore, motivated by [1] and the classical definition of the space of functions of bounded variation for a given m-dimensional open domain, cf [2], we extend such a definition replacing the usual space of test functions by a first order Sobolev space, and the m-dimensional domain by a nonsmooth surface which is embedded in $\mathbb{R}^3$. Finally, we present different definitions for functions of bounded variation on nonsmooth surfaces showing their equivalence.

References

[1] L. Ambrosio and S. Di Marino. Equivalent definitions of BV space and of total variation on metric measure spaces. *Journal of Functional Analysis*, 266 (7), 4150-4188, 2014.

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Fast gradient method with dynamic smoothness parameter

E. Gorgone[1] , A. Frangioni[2] , B. Gendron[3]

[1]Dipartimento di Matematica e Informatica, Universitá di Cagliari,
[2]Dipartimento di Informatica, Universitá di Pisa, [3]Université de Montreal

We present and computationally evaluate a variant of the fast subgradient method of [1] that is capable of exploiting information, even if approximate, about the optimal value of the problem. This information is available in some applications, among which the computation of bounds for hard Integer Programs. We exploit the information to dynamically change the critical smoothness parameter of the algorithm, showing that this results in a better convergence profile of the algorithm.

References

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Semi-supervised learning using phase field models on graphs and hypergraphs

M. Stoll[1] , J. Bosch[2] , S. Klamt[1]

[1] MPI Magdeburg, [2] University of British Columbia

Diffuse interface methods have recently been introduced for the task of semi-supervised learning. The underlying model is well-known in materials science but was extended to graphs using a Ginzburg-Landau functional and the graph Laplacian. We generalize the previously proposed model by a non-smooth potential function. Additionally, we show that the diffuse interface method can be used for the segmentation of data coming from hypergraphs. For this we show that the graph Laplacian in almost all cases is derived from hypergraph information. Additionally, we show that the formerly introduced hypergraph Laplacian coming from a relaxed optimization problem is well suited to be used within the diffuse interface method. We present computational experiments for graph and hypergraph Laplacians.

CT 5. Robust optimization/Optimization under uncertainty

Wednesday, Time: 13.30-15.30, Room: Hall L1, Building L

Nonlinear Robust Optimization of PDE-Constrained Problems using Second-Order Approximations

P. Kolvenbach[1] , S. Ulbrich[1]

[1]Technische Universität Darmstadt, Fachbereich Mathematik

We present an algorithm for the robust optimization of nonlinear PDE-constrained problems with ellipsoidal uncertainty sets [1, 2]. The uncertainty is addressed with a standard worst-case approach which leads to a min–max formulation. Using second-order approximations of the involved objective and constraint functions, the evaluation of the worst-case functions reduces to solving trust-region subproblems. The worst-case functions can be differentiated except in the trust-region hard case, so that the robust counterpart program can be solved with nonsmooth optimization methods. We address high-dimensional uncertainties with matrix-free methods and nonlinearities of higher order with an optimization-based adjustment of the expansion points of the second-order approximations. We apply our method to the parametrized shape optimization of elastic bodies and present numerical results.

References

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A Smoothing Approach to Chance Constrained Optimization of Parabolic PDE Systems under Uncertainty: A Case-Study

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Parabolic partial differential equations (PDEs) are widely used to describe engineering problems of space- and time-dependent systems. In most practical applications, uncertainties arising from imprecise model parameters and random external influences have to be considered. Optimization and optimal control of state-constrained parabolic PDE systems under uncertainty have not yet been properly studied. This is basically because of difficulties to treat inequality constraints of state variables which are random in nature. In this work, we define state inequality constraints of PDE systems in terms of chance (probabilistic) constraints. Chance constrained optimization problems are inherently non-smooth and extremely difficult to solve by directly using available approaches. We use an inner-outer smoothing approach [1] to set up a parametric optimization problem so as to achieve approximate solutions of a chance constrained PDE optimization (CCPDE) problem. The solutions of the inner approximation are

always feasible to the CCPDE and converge to a solution of the CCPDE with respect to the approximation parameter. Consequently, the inner approximation guarantees a priori feasibility. On the other hand, the feasible set of the outer approximation forms a monotonic decreasing sequence that converges to the feasible set of the CCPDE. Thus, the optimal solutions of the parametric outer approximation problem also converge to the solution of the CCPDE [1]. In this way, the outer approximation serves as a parametric tuning strategy. The viability of our smoothing approach is demonstrated through optimal control of a thermodynamic process. Here, we consider optimal heating of a solid bar with heterogeneous materials. Due to the material heterogeneity, the heat transfer coefficient is a random parameter. Heating energy is to be injected at both ends of the bar in such a way that the temperature in the middle of the bar reaches a desired level, during which a temperature upper bound has to be satisfied over the bar. The objective of the CCPDE is to determine an optimal heating strategy to achieve the desired goal and, at the same time, to hold the temperature constraint with a user-defined reliability (probability) level. Various results will be presented to demonstrate the proposed approach.

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Optimization techniques for parametric differential equations

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Uncertainties in dynamical systems arise in many real world problems of physics, chemistry, biology, economics etc.

A number of schemes have been developed to deal with uncertain parametric differential equations. The scheme presented in [1] has been extended for computing the error bounds for approximate solutions of initial value problems for parametric ordinary differential equations. The theoretical development is adapted to the development of optimization models for parametric systems. These optimization problems are then solved to compute preconditioned defect estimates. Finally using these defect estimates and conditional differential inequalities, error bounds for the solution of parametric IVPs are computed.

The scheme is implemented using MATLAB and AMPL. The resulting enclosures are compared with the existing packages VALENCIA-IVP, VNODE-LP and VSPODE for bounding solutions of parametric ODEs.

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Optimization-Free Robust MPC around the Terminal Region

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Model predictive control (MPC) has become a standard tool for the regulation of dynamical systems with state and input constraints (see, e.g., [1]). Conceptually, the control scheme is based on repeatedly solving finite-time optimal control problems (OCPs) in a receding horizon fashion. MPC is well-established in theory and practice particularly for processes that can accurately be modeled by linear systems. In this talk, we study the predictive control of linear discrete-time systems with additive disturbances. There exist different strategies to cope with disturbances in the framework of MPC. In nominal MPC, disturbances are ignored and intrinsic robustness properties of MPC are exploited (see, e.g., [2]). However, better performance is usually achieved by robust MPC (RMPC) schemes that explicitly take disturbances into account. This talk addresses tube-based RMPC, where robustness is guaranteed by investigating families of potential system trajectories merged into so-called tubes (see [3]). We present a novel dual-mode control scheme that significantly reduces the computational effort compared to existing RMPC approaches. Specifically, we propose a method for the computation of a large set $\mathcal{C}$ on which no OCP needs to be solved online. The novel control scheme then requires the conventional solution of the OCP outside of $\mathcal{C}$. However, for all system states in $\mathcal{C}$, the optimal solution results in a simple linear control law that can easily be computed before runtime of the controller. The method is motivated by the trivial observation that, for nominal MPC, no optimization is required for all system states in the terminal set $\mathcal{T}$ that describes the target set in the OCP. In fact, for nominal MPC, it is well-known that the unconstrained linear-quadratic regulator (LQR) is optimal on $\mathcal{T}$. While this observation cannot be directly transferred to RMPC, we show that suitable sets $\mathcal{C}$ exist in the neighborhood of $\mathcal{T}$ and state an algorithm for their computation. Technically, this algorithm results in a mixed-integer linear program (MILP) that can efficiently be solved using state-of-the-art software (e.g., Gurobi). The approach is illustrated with two examples for which the numerical effort during runtime of the controller can be reduced by up to 46.27% compared to classical tube-based RMPC.

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A Stochastic gradient method for Wasserstein barycenters of distributions

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We present a new algorithm based on stochastic gradient methods to compute barycenter of probabilities distribution, in the sens of Wassertein. We are interested in the fast computation of barycenter of probability distributions, which can used for instance to perform clustering for machine learning. The optimal transport [1] provides a distance to compare two distributions, the Wasserstein distance, recently used in barycenter problems [2]. Fast methods to compute these barycenters are required when handling huge amounts of data. We introduce a algorithm of the stochastic gradient class, based on an entropic regularization of barycenter problem. We prove the convergence rate and give the complexity. We also compare this algorithm to the Sinkhorn iterations for barycenters introduced in 2015 [3]. We present numerical simulations for both methods, applied to a set of distributions corresponding to speed and accelerations of different type of traffic. The simulations confirm that the algorithm adapted is faster for distributions with large support.

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Gradient Method With Inexact Oracle for Composite Non-Convex Optimization

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We introduce a new first-order method for non-convex composite optimization problems with inexact oracle. Namely, our problem of interest is as follows

$$\min_{x \in X \subseteq \mathcal{E}} \{\psi(x) := f(x) + h(x)\}, \tag{1}$$

where X is a closed convex set, $h(x)$ is a simple convex function, e.g. $\|x\|_1$. We assume that $f(x)$ is a general function endowed with an inexact first-order oracle. Informally speaking, at any point we can approximately calculate the value of the function and construct a quadratic function, which approximately bounds our $f(x)$ from above. An example of problem with this kind of inexactness is given in [1], where the authors study a learning problem for parametric PageRank model. We give several examples of such inexactness: smooth non-convex functions with inexact Hölder-continuous gradient, functions given by auxiliary uniformly concave maximization problem, which can be solved only approximately.

Already in [2], the author analyzed how different types of inexactness in gradient values influence gradient method for unconstrained smooth convex problems. At the moment, theory for convex optimization algorithms with inexact oracle is well-developed in a series of papers [3, 4, 5]. In [3], it was proposed to calculate inexactly the gradient of the objective function and extend Fast Gradient Method of [6] to be able to use inexact oracle information. In [4], a general concept of inexact oracle is introduced for convex problems, Primal, Dual and Fast gradient methods are analyzed. In [5], the authors develop Stochastic Intermediate Gradient Method for problems with stochastic inexact oracle, which provides good flexibility for solving convex and strongly convex problems with both deterministic and stochastic inexactness.

The theory for non-convex smooth, non-smooth and stochastic problems is well developed in [7, 8]. In [7], problems of the form (1), where $X \equiv \mathbb{R}^n$ and $f(x)$ is a smooth non-convex function are considered in the case when the gradient of $f(x)$ is exactly available, as well as when it is available through stochastic approximation. Later, in [8] the authors generalized these methods for constrained problems of the form (1) in both deterministic and stochastic settings.

Nevertheless, it seems to us that gradient methods for non-convex optimization problems with deterministic inexact oracle lack sufficient development. The goal of this paper is to fill this gap.

For the introduced class of problems, we propose a gradient-type method, which allows to use different proximal setup to adapt to geometry of the feasible set, adaptively chooses controlled oracle error, allows for inexact proximal mapping. We provide convergence rate for our method in terms of the norm of generalized gradient mapping and show that, in the case of inexact Hölder-continuous gradient, our method is universal with respect to Hölder parameters of the problem. Finally, in a particular case, we show that small value of the norm of generalized gradient mapping at a point means that a necessary condition of local minimum approximately holds at that point.

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CT 6. Optimization methods II

Wednesday, Time: 13.30-15.30, Room: L1.201, Building L

A Mixed Integer Program for the Spanning Distribution Forest Problem

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The problem of partitioning an electrical distributions system into subsections satisfying radial- ity and ampacity constraints is well represented using the graph problem of finding a Spanning Distribution Forest (SDFP) [1]. It is defined for an undirected graph $G = (V, E)$ with a set of nodes V and a set of edges E. The set of nodes V is split into two disjunct subsets V_s and V_d. Each node $u \in V_s$ is called a source node and has a corresponding positive integer value $sup(u)$. Elements of the second subset $v \in V_d$ are demand nodes and have a corresponding positive integer value $dem(v)$. There is a maximal capacity c_e for each edge $e \in E$ and it is a positive integer value. Similar to the standard network flow problem, each source $s \in V_s$ can send at most an amount $sup(s)$ of flow to the demand nodes through edges in E. In the SDFP a node $u \in V_d$ can receive the needed demand $dem(u)$ only from a single source node s. Secondly, the edges in E through which the flow from a source node s meets the demands must be a tree. Each demand node in the tree must have its demand satisfied. Further, the flow through each edge must satisfy the capacity constraint. Let us define a flow tree $T_s \subset G$ as a tree having a single source node s and satisfying the previously mentioned constraints. The goal of the SDFP is to find a set of vertex disjoint flow trees $\Pi = \{T_1, T_2, \ldots, T_n\}$ of the graph G that satisfy the maximal demand. In the parametric version of the SDFP there is an additional constraint that all demands are satisfied. To be more precise, since covering all the demands is not possible for each configuration, a new parameter $0 < r \leq 1$ is used to scale all the demands to $r \cdot dem(u)$ for which this is possible. The objective is to find the maximal value of r. In our previous work [2] we have developed an integer model and efficient pre-processing procedure for the closely related problem of maximal partitioning of graphs with supply and demand. In this talk, we present an extension of this work to the SDFP. Further, with the objective of finding near optimal solutions for large scale problem instances, we propose a matheuristic approach which uses the proposed integer program to locally improve the solution. More pre- cisely, the improvement is performed to subgraphs, selected using a heuristic procedure, on which pre-processing is especially efficient.

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Maximizing wireless directional sensor networks lifetime. A Lagrangian relaxation approach

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Wireless sensor networks are more and more applied in several fields such as security, climate monitoring and risk detection. We present a Lagrangian relaxation approach for the Directional Sensor Network Lifetime Problem (DSNLP). Given a certain area to be monitored, the DSNLP problem aims at providing the best activation scheduling for a set of sensors, which are supposed to be directional and hence characterized by a discrete set of possible radii and aperture angles. The directionality of the sensors requires also that, for each of them, the orientation axis has to be scheduled as well. Such problem has been studied in literature both by adopting heuristic approaches as well as column generation based models. We propose an alternative mixed integer nonlinear formulation which is suitable for a Lagrangian type decomposition. We design an ad hoc dual ascent procedure, coupled with a heuristics. The results obtained by implementing our methods are also presented.

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Transportation management: methods and algorithms for solution

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A dilemma often facing motorists is to choose which route should be taken to realise the fastest (i.e. shortest) journey time. The route choices between an origin and destination might typically be a direct main road, a shorter route through narrow side streets in residential areas, or a longer but (potentially) faster journey using a nearby motorway or ring-road.

In the absence of effective traffic control measures, an approximate equilibrium travel time results between the routes available. However, this may not be optimal, as faster overall journey times may have resulted had car drivers allocated themselves differently between the routes. In what has become known as the Braess Paradox [1], this difference between equilibrium and optimal travel times can lead to the decidedly counter-intuitive result, that additions to road capacity, typically through road construction, lead to slower - not faster - car journey times.

Here a transportation model where journey time is the only criteria for route choice is solved by dynamic programming, genetic algorithms and numerical or analytical techniques. Results show close links between solutions of certain (discrete and continuous)

optimization and equilibrium problems [3]. The effects induced by closing certain road segments are also investigated.

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An Elastic Primal Active Set Method for Large QPs

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We consider SQP methods for large structured NLPs that may arise from branch-and-bound schemes for MINLPs. The talk presents a generic primal active set method that employs an arbitrary, possibly "matrix-free" KKT solver in ℓ_1 and ℓ_2 slack relaxations of the QP subproblems to allow for efficient warm starts from infeasible points. Our approach involves Schur complement and projection techniques that preserve the NLP sparse structure in the KKT system. We also discuss relevant aspects of the software design and present numerical results for QPs from the CUTEst test set and for large structured QPs.

Rigorous n unit squares packing

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The n unit squares packing problem asks for the side length s_n of the smallest square(the container) that covers n unit squares free to rotate and with no intersecting interior. Friedman [3] maintains a dynamic survey with the proof of all optimal values found so far as well as tables with updated lower and upper bounds. In particular, lower and upper bounds on s_n are known for $n \leq 85$ and $n \leq 100$ respectively. Optimal values are known only for $n \in \{2, 3, 5, 6, 7, 8, 10, 14, 15, 24, 35\}$ and for trivial cases where n is a square.

A set $\mathcal{S}$ of points in the interior of the container is called non-avoidable if each square in the container contains at least one point from $\mathcal{S}$. The set $\mathcal{S}$ is called strictly non-avoidable if each square in the container contains at least one point from $\mathcal{S}$ in its interior. The method to determine lower bounds and the optimal value for several cases relies on non-avoidance lemmas [3]. An important strictly non-avoidance lemma states that if a triangle with all side lengths lower than one contains the center of a unit square, then the square contains at least one vertex of the triangle in its interior. Therefore all vertices of a triangulation covering a container with triangles of side length smaller than one is strictly non-avoidable.

We propose a computer-assisted procedure to rigorously enclose optimal configurations for the unit squares packing problem. Our method embeds strictly non-avoidance lemmas into an in- terval branch and bound framework. Unlike other results from the literature, the new approach does not depend on the number of packed squares. It is also constructive, providing an optimal configuration at the end of the process.

We write the n unit squares problem as a constraint satisfaction problem. Given a configuration for the unit squares problem, we check its feasibility with rigorous verification methods, and once we find a verified feasible solution for the CSP, the next step is to prove its optimality. To do so, we add to the CSP a constraint stating that any a new solution must be at least as good as the current one. Since the resulting CSP is too complicated, we need to include more constraints to the problem to make the computations viable and therefore we use the strictly non-avoidance lemma to split the original CSP into a set of $\binom{K}{n}$ subproblems where K is the number of vertices in the triangulation.

We use symmetry arguments to reduce the number of subproblems that need to be solved by the interval branch and bound algorithm. If a CSP provides a new solution then we restart the process. Otherwise, the interval branch and bound will return the empty solution for all subproblems except the one with the current solution. In this case, the procedure will rigorously enclose the current solution and claims that it is the optimal one. For example, we found a feasible configuration for the 5-unit squares packing problem that gives a non-avoidable set of 13 points. Based on this, 1287 subproblems must be analyzed. Applying symmetry arguments, we reduce the number of subproblems to 90. If the unique solutions for these 90 problems is the current configuration it must be optimal.

We implement the new approach into the *GloptLab* [2, 1] environment that provides state-of-the-art tools for rigorous computing.

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Author Index

Adams, C., 63
Annunziato, M., 29
Apkarian, P., 15
Astorino, A., 80
Atolia, M., 31
Auriemma, S., 66
Büskens, C., 24, 32, 68
Bagdasar, O., 60, 80
Bagirov, A.M., 71
Bajović, D., 34
Banovic, M., 66
Barlow, J., 48
Beermann, D., 64
Benning, M., 58
Berry, S., 80
Bigi, G., 40
Bonalli, R., 33
Bonettini, S., 52
Bortolotti, V., 50
Borzì, A., 29
Bosch, J., 73
Boyaval, S., 42
Bredies, K., 57
Bucher, M., 39
Buchheim, C., 17
Caillau, J.-B., 54, 56
Chen, K., 49
Chouzenoux, E., 51
Chudej, K., 25
Coli, V. L., 47
Conforti, M., 17
Corsaro, S., 35
Dür, M., 63
Dellnitz, M., 64
Dinh, T.P., 72
di Serafino, D., 19, 48
Dostál, Z., 49
Dvurechensky, P., 77
Ehrhardt, M. J., 58
Féjoz, J., 54
Faßbender, H., 20
Fabrini, G., 43
Fazal, Q., 75
Fischer, A., 25, 35, 38
Flaßkamp, K., 24, 32
Fleig, A., 27
Forets, M., 45
Frangioni, A., 73
Frerick, L., 63
Günther, C., 62
Gathungu, D., 29
Gaudioso, M., 71, 80
Geletu, A., 74
Gendron, B., 73
Gerdts, M., 23
Giallombardo, G., 71
Gondzio, J., 14
Gorgone, E., 73
Grüne, L., 27
Gräßle, C., 44
Grad, S.-M., 61
Grepl, M., 42
Guglielmi, R., 27
Hérissé, B., 33
Hajian, S., 65
Hante, F. M., 28
Harbrecht, H., 18
Hegerhorst, L., 71
Henrion, D., 45
Herrich, M., 38
Herrmann, M., 58, 72
Herzog, R., 58, 72
Hintermüller, M., 65
Hinze, M., 44
Hiriart-Urruty, J.-B., 69
Iapichino, L., 43
Jakovetić, D., 34
Jando, D., 42
Josz, C., 46
Jovanovic, R., 79
Jurgelucks, B., 67
Kärcher, M., 42
Kanzow, C., 34, 38
Karl, V., 38
Kimiaei, M., 69
Kimmerle, S.-J., 23
Klamt, S., 73
Kolo, K., 28
Kolvenbach, P., 74
Kröner, H., 58
Krejić, N., 34
Krklec Jerinkić, N., 34
Kuřátko, J., 68
Kuhlmann, R., 17, 68
Kuhn, D., 16
Kulshreshtha, K., 70
Kunisch, K., 16
Löbhard, C., 65
Lampariello, L., 39
Landi, G., 50

Lasserre, J. B., 46
Le Rhun, A., 77
Lorenz, D., 20
Loungani, P., 31
Luig, K., 35
Müller, J.D., 66
Mérigot, Q., 15
MacIntyre, K., 70
Magron, V., 44, 45
Maierhofer, G., 58
Marino, Z., 35
Markót, M., 81
Marnissi, Y., 51
Maurer, H., 31, 33
Meyer, C., 17
Miglionico, G., 71, 80
Molzahn, D. K., 46
Montanher, T., 81
Moretto, E., 61
Morini, B., 35
Morotti, E., 47
Narayanan, S.H.K., 70
Neumaier, A., 69, 75, 81
Nguyen, T.M.T., 72
Nyenyezi, J. B., 21
Orban, D., 19
Orieux, M., 54
Pearson, J., 20
Peitz, S., 64
Pesquet, J.-C., 51
Piccolomini, E. L., 47
Pickenhain, S., 28
Poggiolini, L., 54
Pomet, J.-B., 56
Popovici, N., 60–62, 80
Porcelli, M., 35
Potschka, A., 65
Prato, M., 52
Rebegoldi, S., 52
Rose, D., 81
Rouot, J., 56
Rozza, G., 16
Runge, M., 32
Sagratella, S., 39, 40
Sartenaer, A., 21
Sass, J., 37
Schäfer, K., 32
Schönlieb, C. B., 17, 58
Scharmacher, N., 44
Schmidt, P., 74
Schmidt, S., 58, 72
Schneider, C., 24
Schulze Darup, M., 76
Schumann, E., 37
Schwartz, A., 15, 39
Sciandrone, M., 14
Selassie, W., 74
Semmler, W., 31
Sherry, F., 58
Spencer, J., 49
Steck, D., 38
Stefani, G., 54
Steinbach, M., 81
Stoll, M., 73
Strasdat, N., 35
Sun, H., 57
Tardella, F., 36
Thi, H.A.L., 72
Thies, T., 35
Toraldo, G., 48
Trélat, E., 33
Ulbrich, S., 64, 74
Valkonen, T., 56
Veroy, K., 42
Vestweber, L., 20
Vidal, J., 58, 72
Viola, M., 48
Vlach, O., 49
Volkwein, S., 43, 64
Voss, S., 79
Wachsmuth, D., 38
Wachsmuth, G., 24
Walter, A., 64
Walther, A., 66
Weber, F., 35
Weisser, T., 46
Worthmann, K., 26
Yücel, H., 67
Zama, F., 50
Zanni, L., 47
Zheng, Y., 51

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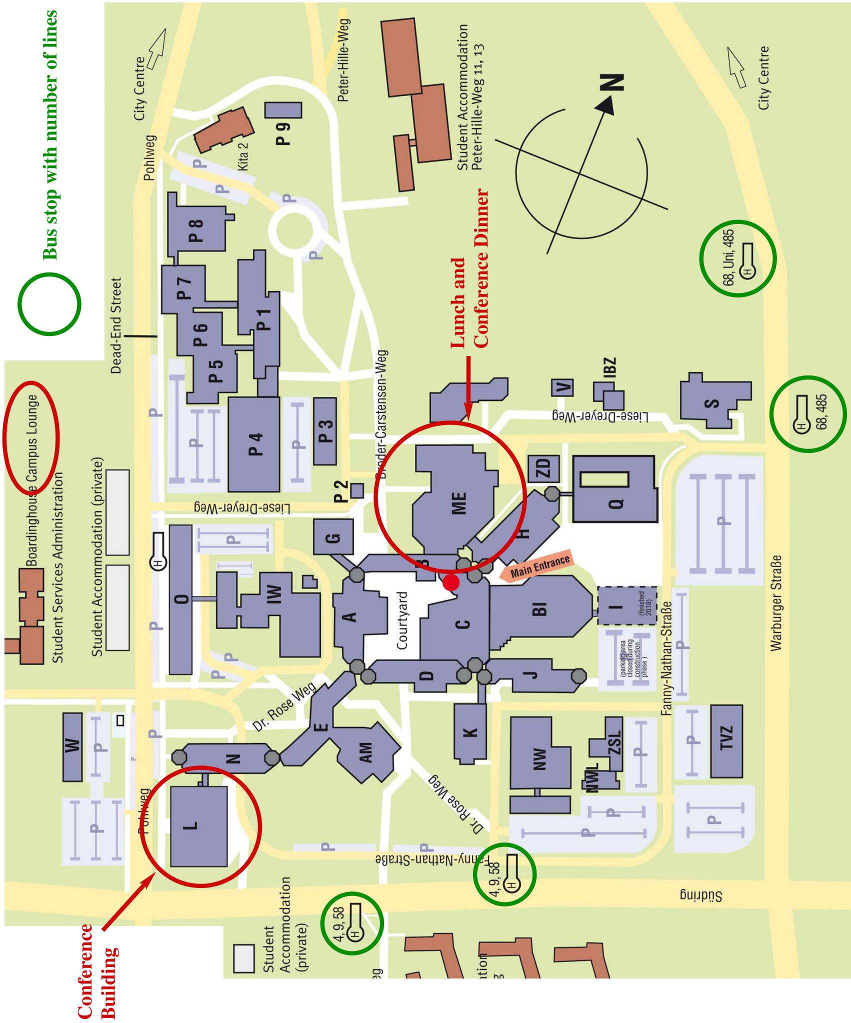

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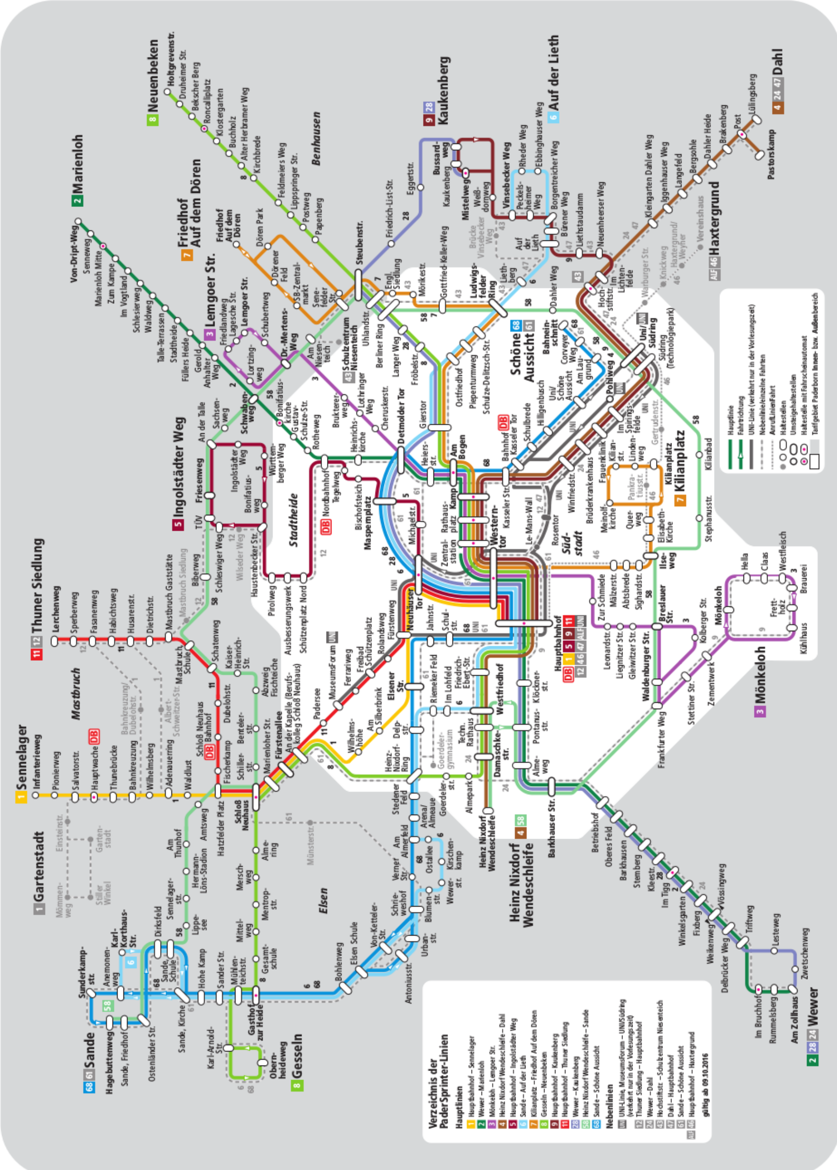

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