Dealing with Optimization Problems
Constrained by Nonlinear Non-Smooth PDEs

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Outline

1. Motivation & Model Problem
2. Optimization Strategy for Non-Smooth Problems
3. Numerical Results
4. Conclusion and Outlook
Motivation

- **Aim:** simulation and optimization of hysteresis
- **Challenge:**
  - Switches
  - History dependency
- **Study model problem:** PDE with switches
Model Problem for Switches

\[
\min_{y, u \in H_0^1 \times L^2} \frac{1}{2} \| y - y_d \|_{L^2}^2 + \frac{\alpha}{2} \| u \|_{L^2}^2
\]

s.t. \quad - \Delta y + \ell(y) - u = 0 \quad \text{in} \ \Omega

- Lipschitz domain \( \Omega \subset \mathbb{R}^n, n \in \mathbb{N} \)
- \( \ell : H_0^1(\Omega) \to L^2(\Omega) \) non-smooth, locally Lipschitz continuous, bounded and strictly monotone
- Non-differentiabilities are constructed from \( \text{abs}() \) and smooth elementals. (Also covers \( \text{min}() \) and \( \text{max}() \)).
Model Problem for Switches

\[
\min_{(y,u) \in H^1_0 \times L^2} \frac{1}{2} \|y - y_d\|_{L^2}^2 + \frac{\alpha}{2} \|u\|_{L^2}^2
\]

s.t. \[ -\Delta y + \ell(y) - u = 0 \text{ in } \Omega \]

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- Non-differentiablilities are constructed from abs() and smooth elementals. (Also covers min() and max()).
Model Problem for Switches

\[
\min_{(y,u) \in H^1_0 \times L^2} \frac{1}{2} \| y - y_d \|_{L^2}^2 + \frac{\alpha}{2} \| u \|_{L^2}^2 \\
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- \( \ell : H^1_0(\Omega) \to L^2(\Omega) \) non-smooth, locally Lipschitz continuous, bounded and strictly monotone
- Non-differentiablilities are constructed from \( \text{abs}() \) and smooth elementals. (Also covers \( \text{min}() \) and \( \text{max}() \)).

Challenge: Standard techniques can not be applied.
Observed Properties

- Weakly lower semi-continuous and twice continuously Fréchet-differentiable objective functional

- Well posed simulation problem with unique solution \( y \) for any given control \( u \)

  Browder-Minty Theorem

- Well defined, Lipschitz continuous and directionally differentiable control-to-state-mapping \( S : u \mapsto y \)

- Existence of a solution for the optimal control problem
Previous Results for Finite Dimensions
Previous Results for Finite Dimensions

Theoretical results for

- Lipschitz continuous, piecewise smooth function $f : \mathbb{R}^n \to \mathbb{R}$
- All non-differentiabilities are incorporated by $\text{abs()}$

$\Rightarrow$ First and second order optimality conditions

Development of minimization algorithm

- Convergence theory & promising numerical results

$\Rightarrow$ LiPsMin - Lipschitzian Piecewise Smooth Minimization
Handling of abs

Convert max and min to abs via:

\[
\begin{align*}
\max(u, v) &= \frac{1}{2}(u + v + |v - u|) \\
\min(u, v) &= \frac{1}{2}(u + v - |v - u|)
\end{align*}
\]

| \(\ell(y)\) | \(= \Psi(y, |y|)\) |
|---|---|
| \(z_i\) | \(= \psi^i(y, \sigma z)_{j<i}\) |
| \(\sigma_i\) | \(= \text{sgn}(z_i)\) |
| \(\sigma_i z_i\) | \(= \text{abs}(z_i)\) |

\[
\hat{\ell}(y, \sigma z) = \psi^{s+1}(y, \sigma z)_{j<s+1}, \quad \sigma z = (\sigma_1 z_1, \ldots, \sigma_s z_s)
\]

**Table:** Reduced evaluation procedure
Example

Reduced evaluation procedure for
\[ \ell(y) = \max(0, y) + \min(0, y + \frac{1}{2}|y|) \]

| \( \ell(y) \) | \( = y + \frac{3}{4}|y| - \frac{1}{2}|y + \frac{1}{2}|y|| \) |
|----------------|--------------------------------------------------|
| \( z_1 \)     | \( = \psi^1(y, \sigma z)_{j \prec 1} = y \)     |
| \( \sigma_1 \) | \( = \text{sgn}(z_1) \)                         |
| \( \sigma_1 z_1 \) | \( = \text{abs}(z_1) \)                     |
| \( z_2 \)     | \( = \psi^2(y, \sigma z)_{j \prec 2} = y + \frac{1}{2}\sigma_1 z_1 \) |
| \( \sigma_2 \) | \( = \text{sgn}(z_2) \)                         |
| \( \sigma_2 z_2 \) | \( = \text{abs}(z_2) \)                     |
| \( \hat{\ell}(y, \sigma z) \) | \( = \psi^3(y, \sigma z)_{j \prec 3} = y + \frac{3}{4}\sigma_1 z_1 - \frac{1}{2}\sigma_2 z_2 \) |
The Idea of Abs-Linearization in Finite Dimensions

\[
\min_{x_1, x_2} \max(0, x_2^2 - \max(0, x_1))
\]
\[
\min_{x_1, x_2} \max(0, x_2^2 - \max(0, x_1))
\]

\[
= \frac{1}{2} [x_2^2 - \frac{1}{2} (x_1 + |x_1|)]
+ \frac{1}{2} [\mid x_2^2 - \frac{1}{2} (x_1 + |x_1|)]
\]

1. Substitute all abs:

z_1 \leftarrow x_1
z_2 \leftarrow x_2^2 \quad \min z_1, z_2

s.t. z_1 - x_1 = 0
\quad z_2 - x_2 + \frac{1}{2} (x_1 + |x_1|) = 0
\[
\begin{align*}
\min_{x_1, x_2} & \max(0, x_2^2 - \max(0, x_1)) \\
& = \frac{1}{2} [x_2^2 - \frac{1}{2}(x_1 + |x_1|)] \\
& + \frac{1}{2} [|x_2^2 - \frac{1}{2}(x_1 + |x_1|)|]
\end{align*}
\]

1. Substitute all abs:

\[
\begin{align*}
z_1 & \leftarrow x_1 \\
z_2 & \leftarrow x_2^2 - \frac{1}{2}(x_1 + \text{sgn}(z_1)z_1)
\end{align*}
\]
\[
\begin{align*}
\min_{x_1, x_2} & \max(0, x_2^2 - \max(0, x_1)) \\
& = \frac{1}{2} [x_2^2 - \frac{1}{2} (x_1 + |x_1|)] \\
& + \frac{1}{2} [|x_2^2 - \frac{1}{2} (x_1 + |x_1|)|]
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1. Substitute all abs:

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z_1 & \leftarrow x_1 \\
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\end{align*}
\]

2. Introduce new variables \(\sigma_i\) for \(\text{sgn}(z_i)\):
\[
\min_{x_1, x_2} \max(0, x_2^2 - \max(0, x_1)) \\
= \frac{1}{2} [x_2^2 - \frac{1}{2} (x_1 + |x_1|)] \\
+ \frac{1}{2} [|x_2^2 - \frac{1}{2} (x_1 + |x_1|)]
\]

1. Substitute all abs:

\[z_1 \leftarrow x_1\]
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2. Introduce new variables \(\sigma_i\) for \(\text{sgn}(z_i)\):

\[
\min_{z_1, z_2} \frac{1}{2} [z_2 + \sigma_2 z_2] \\
s.t. \quad z_1 - x_1 = 0 \\
\quad z_2 - x_2^2 + \frac{1}{2} (x_1 + \sigma_1 z_1) = 0
\]
\[
\min_{x_1, x_2} \max(0, x_2^2 - \max(0, y_1))
\]

\[
\begin{align*}
\min_{z_1, z_2} & \quad \frac{1}{2} [z_2 + \sigma_2 z_2] \\
\text{s.t.} & \quad z_1 - x_1 = 0 \\
& \quad z_2 - x_2^2 + \frac{1}{2} (x_1 + \sigma_1 z_1) = 0 \\
& \quad \sigma_1 z_1 \geq 0 \\
& \quad \sigma_2 z_2 \geq 0
\end{align*}
\]

- New problem is smooth if \( \sigma \) is fixed parameter
- Changing a \( \sigma_i \), means changing a region
$\mathbb{R}^n$ vs. Function Space Scenario

- $\mathbb{R}^n$
  - $\sigma_i \in \{-1, 0, 1\}$
  - Piecewise linearization
  - Decomposition of the domain

- Function Space
  - Need adopted ideas and new concepts.
## Handling of abs in Function Spaces

\[
\ell(y) = \psi(y, |y|)
\]

| \(z_i\) | \(\psi^i(y, \sigma z)_{j \prec i}\) |
| \(\sigma_i\) | \(\text{sgn}(z_i)\) |
| \(\sigma_i z_i\) | \(\text{abs}(z_i)\) |

\[
\hat{\ell}(y, \sigma z) = \psi^{s+1}(y, \sigma z)_{j \prec s+1}, \quad \sigma z = (\sigma_1 z_1, \ldots, \sigma_s z_s)
\]

**Nemytskii operator**

\[
\text{abs} : H^1_0(\Omega) \rightarrow L^2(\Omega), \quad [\text{abs}(y)](x) = |y(x)| \quad \text{f.a.a. } x \in \Omega
\]

\[
\sigma_i : H^1_0(\Omega) \rightarrow L^2(\Omega), \quad [\sigma_i(z)](x) = \text{sgn}(z_i(x))z_i(x) \quad \text{f.a.a. } x \in \Omega
\]

\[
\Rightarrow \sigma_i(z_i) = \text{abs}(z_i) \in L^2(\Omega)
\]
Equivalent Problem Reformulation

\[
\begin{align*}
\min_{(y,u) \in H^1_0 \times L^2} & \quad \frac{1}{2} \| y - y_d \|_{L^2}^2 + \frac{\alpha}{2} \| u \|_{L^2}^2 \
\text{s.t.} & \quad - \Delta y + \ell(y) - u = 0
\end{align*}
\]

\[
\begin{align*}
\min_{(y,u) \in H^1_0 \times L^2} & \quad J(y,u) \\
\text{s.t.} & \quad - \Delta y + \hat{\ell}(y,\sigma z) - u = 0 \\
& \quad \psi^i(y,\sigma z)_{j < i} - z_i = 0 \\
& \quad \sigma_i z_i - \text{abs}(z_i) = 0 \quad \forall \ i = 1, \ldots, s
\end{align*}
\]
Decomposition into Smooth Branch Problems

\[
\begin{align*}
\min_{(y,u) \in H^1_0 \times L^2} \quad & \frac{1}{2} \| y - y_d \|_{L^2}^2 + \frac{\alpha}{2} \| u \|_{L^2}^2 \\
\text{s.t.} \quad & - \Delta y + \ell(y) - u = 0
\end{align*}
\]

\[
\downarrow
\]

\[
\begin{align*}
\min_{(y,u) \in H^1_0 \times L^2} \quad & \mathcal{J}(y,u) \\
\text{s.t.} \quad & - \Delta y + \hat{\ell}(y, \sigma z) - u = 0 \\
& \psi^i(y, \sigma z)_{j \prec i} - z_i = 0 \quad (1) \\
& \sigma_i z_i \geq 0 \quad \forall \ i = 1, \ldots, s \\
& \sigma_i : \Omega \to \{-1, +1\}
\end{align*}
\]
Optimizations Strategy for Non-smooth Problems

Optimization Strategy - Lagrangian with Penalty

\[
\min_{(y,u)\in H^1_0 \times L^2} \mathcal{J}(y,u)
\]

s.t. \( -\Delta y + \hat{\ell}(y,\sigma z) - u = 0 \)

\[
\psi^i(y,\sigma z)_{j\prec i} - z_i = 0 \quad \forall i = 1,\ldots,s
\]

\[
\sigma_i \geq 0 \quad \forall i = 1,\ldots,s
\]

\[
\sigma_i : \Omega \to \{-1, 1\}
\]

\[
\mathcal{L}(y, u, \sigma z, \lambda_{PDE}, \lambda_1, \ldots, \lambda_s)
\]

\[
= \mathcal{J}(y, u) + (\nabla \lambda_{PDE}, \nabla y)_{L^2(\Omega)} + (\lambda_{PDE}, \hat{\ell}(y, \sigma z) - u)_{L^2(\Omega)}
\]

\[
+ \sum_{i=1, j\prec i}^s (\lambda_i, \psi^i(y, \sigma z)_{j\prec i} - z_i)_{L^2(\Omega)} + \mu \int_\Omega \sum_{i=1}^s \left( \max(-\sigma_i z_i, 0) \right)^4 \, d\Omega
\]

with \( \mu > 0 \) sufficiently large.
Necessary Optimality Conditions

\[ 0 \overset{!}{=} \frac{\partial J}{\partial y} \tilde{y} + (\Delta \lambda_{\text{PDE}}, \Delta \tilde{y}) + (\lambda_{\text{PDE}}, \frac{\partial \hat{\ell}}{\partial y} \tilde{y}) + \sum_{i=1}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z_j)}{\partial y} \tilde{y}) \quad \forall \tilde{y} \]

\[ 0 \overset{!}{=} \frac{\partial J}{\partial u} \tilde{u} - (\lambda_{\text{PDE}}, \tilde{u}) \quad \forall \tilde{u} \]

\[ 0 \overset{!}{=} (\Delta \tilde{\lambda}_{\text{PDE}}, \Delta \tilde{y}) + (\tilde{\lambda}_{\text{PDE}}, \hat{\ell}) - (\tilde{\lambda}_{\text{PDE}}, u) \quad \forall \tilde{\lambda}_{\text{PDE}} \]

\[ 0 \overset{!}{=} (\tilde{\lambda}_i, \psi^i(y, \sigma z)_{j < i} - z_i)_{L^2(\Omega)} \quad \forall \tilde{\lambda}_i, \quad \forall i = 1, \ldots, s \]

\[ 0 \overset{!}{=} (\lambda_{\text{PDE}}, \frac{\partial \hat{\ell}(y, \sigma z)}{\partial z_k} \tilde{z}_k) + (\lambda_k, \frac{\partial \psi^i(y, \sigma z_j)}{\partial z_k} \tilde{z}_k) - (\lambda_k, \tilde{z}_k) \]

\[ + \sum_{i=k+1, \ j < i}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z)}{\partial z_k} \tilde{z}_k) + \mu \int_{\Omega} -4\sigma_k \max(-\sigma_k z_k, 0)^3 \tilde{z}_k \ d\Omega \]

\[ \forall \tilde{z}_k, \quad \forall k = 1, \ldots, s \]
Algorithmic Idea

Solving non-smooth optimal control problems:

0. Reformulate non-smoothness in terms of abs.

1. Choose corresponding branch problem, i.e. choose \( \sigma_i : \Omega \rightarrow \{-1, +1\} \).

2. Solve smooth branch problem (e.g. with FEM & Newton) for \((y, u, z, \lambda)\).

3. If \( \lambda_i = 0 \), \( \forall i \): STOP
   Else: Determine new \( \sigma_i \) according to \( \lambda_i \) & go to 1.
How to Switch Branch Problems?

0 \overset{!}{=} \frac{\partial J}{\partial y} \tilde{y} + (\Delta \lambda_{PDE}, \Delta \tilde{y}) + (\lambda_{PDE}, \frac{\partial \hat{\ell}}{\partial y} \tilde{y}) + \sum_{i=1}^{s} (\lambda_i, \frac{\partial \psi_i(y, \sigma_j z_j)}{\partial y} \tilde{y}) \quad \forall \tilde{y}

0 \overset{!}{=} \frac{\partial J}{\partial u} \tilde{u} - (\lambda_{PDE}, \tilde{u}) \quad \forall \tilde{u}

0 \overset{!}{=} (\Delta \tilde{\lambda}_{PDE}, \Delta y) + (\tilde{\lambda}_{PDE}, \hat{\ell}) - (\tilde{\lambda}_{PDE}, u) \quad \forall \tilde{\lambda}_{PDE}

0 \overset{!}{=} (\tilde{\lambda}_i, \psi^i(y, \sigma z)_{j \prec i} - z_i)_{L^2(\Omega)} \quad \forall \tilde{\lambda}_i, \forall i = 1, \ldots, s

0 \overset{!}{=} (\lambda_{PDE}, \frac{\partial \hat{\ell}(y, \sigma z)}{\partial z_k} \tilde{z}_k) + (\lambda_k, \frac{\partial \psi^i(y, \sigma z)_{j \prec i}}{\partial z_k} \tilde{z}_k) - (\lambda_k, \tilde{z}_k)

+ \sum_{i=k+1, j \prec i}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z)_{j \prec i}}{\partial z_k} \tilde{z}_k) + \mu \int_{\Omega} -4\sigma_k \max(-\sigma_k z_k, 0)^3 \tilde{z}_k \ d\Omega \quad \forall \tilde{z}_k, \forall k = 1, \ldots, s
How to Switch Branch Problems?

\[ 0 \leq \mu \int_{\Omega} 4 \max(-\sigma_k z_k, 0)^3 \tilde{z}_k \, d\Omega = (\lambda_{PDE}, \frac{\partial \hat{\ell}(y, \sigma z)}{\partial z_k} \tilde{z}_k) + (\lambda_k, \frac{\partial \psi^i(y, \sigma z)}{\partial z_k} \tilde{z}_k) \]

\[ - (\lambda_k \sigma_k, \tilde{z}_k) + \sum_{i=k+1}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z)}{\partial z_k} \tilde{z}_k) \]

\[ \forall k = 1, \ldots, s \]
How to Switch Branch Problems?

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\[ - (\lambda_k \sigma_k, \tilde{z}_k) + \sum_{i=k+1}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z_j)}{\partial z_k} \tilde{z}_k) \]

\[ \forall k = 1, \ldots, s \]

This optimality condition is violated iff \( \exists k \in \{1, \ldots, s\} \) such that \( \sigma_k = \text{sgn}(\lambda_k) \) with

\[ 0 > (\lambda_{PDE}, \frac{\partial \hat{\ell}(y, \sigma z)}{\partial z_k} \tilde{z}_k) + (|\lambda_k|, \frac{\partial \psi^i(y, \sigma z)}{\partial z_k} \tilde{z}_k - \tilde{z}_k) + \sum_{i=k+1}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma z_j)}{\partial z_k} \tilde{z}_k) \]
How to Switch Branch Problems?

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\[ \exists k \in \{1, \ldots, s\} \text{ such that } \sigma_k = \text{sgn}(\lambda_k) \text{ with} \]

\[
0 > (\lambda_{PDE}, \frac{\partial \hat{l}(y, \sigma z)}{\partial z_k} \tilde{z}_k) + (|\lambda_k|, \frac{\partial \tilde{\psi}^i(y, \sigma z)}{\partial z_k} \tilde{z}_k - \tilde{z}_k) + \sum_{i=k+1}^{s} (\lambda_i, \frac{\partial \psi^i(y, \sigma_j z_j)}{\partial z_k} \tilde{z}_k)
\]

\[ \Rightarrow \] Natural strategy: choose k for which RHS is minimal.

\[ \Rightarrow \] Swap \( \sigma_k \) where discretized \( \lambda^h_k \) is pointwise largest.
\[ \Rightarrow \] Explore branch with highest importance first.

\[ \Rightarrow \] Use \( \lambda^h_k \) as indicator to swap sign.

Numerical Results

\[ \ell(y) = \max(0, y) + \min(0, y + \frac{1}{2}|y|), \quad \bar{\Omega} = [0, 1] \times [0, 1] \]

\[ y_d(x_1, x_2) = \min \left( \max \left( |x_1 - \frac{1}{2}|, |x_2 - \frac{1}{2}| \right) - \frac{1}{4}, 0 \right) \]
**Numerical Results**

\[
\ell(y) = \max(0, y) + \min(0, y + \frac{1}{2}|y|), \quad \widetilde{\Omega} = [0, 1] \times [0, 1]
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\[
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Numerical Results

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Numerical Results

\[
\ell(y) = \max(0, y) + \min(0, y + \frac{1}{2}|y|), \quad \Omega = [0,1] \times [0,1]
\]

\[
y_d(x_1, x_2) = \min \left( \max \left( \left| x_1 - \frac{1}{2} \right|, \left| x_2 - \frac{1}{2} \right| \right) - \frac{1}{4}, 0 \right)
\]
Numerical Results

Switching branch problem according to $\lambda_i$

Numerical Results

Switching branch problem according to $\lambda_i$

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Switching branch problem according to $\lambda_i$

Numerical Results

Switching branch problem according to $\lambda_i$
Conclusion

- Optimization of non-smooth Lipschitz-continuous problems
- Extension to function space setting
- Handling and solving without regularizations for the non-smoothness
- Each branch problem is smooth, Newton/SQP allows quick solution
- Stopping criterion: sufficient reduction in adjoint, thus no more switching
Ongoing and Future Work

- **Two publications in preparation**
  - O. Ebel, S. Schmidt, A. Walther, A. Griewank, Optimization by Successive Abs-Linearization in Function Spaces, 2018
  - O. Ebel, S. Schmidt, A. Walther, Solving Non-Smooth Semi-Linear Elliptic Optimal Control Problems with Abs-Linearization, 2018

- **Extension to non-smooth parabolic and hyperbolic PDE constraints**

- **Extension to general switches and hysteresis**

- **Handling non-Lipschitz continuous functions in the problem formulation**

Thank you for your attention!
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