56th session of Seminar Sophus Lie

Universität Paderborn
February 14-15, 2020

Organizing Committee

Kai-Uwe Bux (Universität Bielefeld)
Helge Glöckner (Universität Paderborn)
Joachim Hilgert (Universität Paderborn)
Tobias Weich (Universität Paderborn)

Conference Venue

Institut für Mathematik, Universität Paderborn,
Warburger Straße 100, D-33098 Paderborn
Building O

This conference is financially supported by the Deutsche Forschungsgemeinschaft (DFG)

Contents

General information

Conference venue: Building O

Check-in for Conference: Room O1.252

All talks are held in lecture room O2.
The rooms are equipped with blackboards and a projector.

Coffee breaks take place in Room O1.252.

Internet Access

WIFI access is provided via Eduroam.

Transport

It is possible to reach the University Campus from the city center by foot in roughly 30 minutes or alternatively by bus in about 15 minutes. Tickets can be purchased on the bus (only cash) and at many bus stations (including Hauptbahnhof).

There are three lines going to the University campus from Paderborn train station (Hauptbahnhof) through the city center: line 4, line 9 and line 68. Busses 4 and 9 are running every 15 minutes.

Bus stops close to the campus:

1. Bus stop **Uni-Südring** can be reached from downtown (Hauptbahnhof, Westerntor, Rathausplatz, Kamp) by line 4 (direction Dahl) and line 9 (direction Kaukenberg).

2. Bus stop **Schöne Aussicht** can be reached from downtown (Hauptbahnhof, Neuhäuser Tor, Detmolder Tor, Am Bogen) by line 68, direction **Schöne Aussicht**. It operates every 30 minutes.

Notice: The line **UNI** does not operate during the semester break.

ATM on the Campus

There is an ATM in the Mensa building (ground floor).

Social program

Friday 5:30pm Excursion to Heinz-Nixdorf-Museumsforum

We invite all participants of the conference to a joint excursion to the **Heinz-Nixdorf-Museumsforum** on Friday, July 14. The excursion will include a guided tour to the world's biggest computer museum.

There will be a bus shuttle to the museum departing in front of the conference building at 5pm. More precise information on the departure of the bus will be announced during the conference and on the conference webpage.

Museum's homepage:
`https://www.hnf.de/en/the-hnf.html`

Friday 7:30pm Conference Dinner

The Conference Dinner will be held in the **Schützenhof**, Schützenplatz 1, 33102 Paderborn. The Schützenhof is at 10 minutes walking distance from the Heinz-Nixdorf-Museumsforum.

Schedule

Friday, February 14

8:30 – 8:50	**Registration** (room O1.252)
8:50 – 9:00	**Opening** (Lecture room O2)
9:00 – 9:45	Yves Benoist *Tempered homogeneous spaces*
9:50 – 10:35	Job Kuit *Discrete series representations for real spherical spaces*
10:35 – 11:10	Coffee/Tea
11:10 – 11:55	Erik van den Ban *Fourier inversion of Whittaker functions*
12:00 – 12:30	Ilka Agricola *Holonomy properties of Riemannian homogeneous spaces*
12:30 – 13:45	Lunch
13:45 – 14:30	Armin Rainer *ODE-closed function spaces and groups of diffeomorphisms on* $\mathbb{R}^d$
14:35 – 15:00	Clemens Weiske *Unitary branching laws for real reductive groups*
15:05 – 15:30	Daniel Oeh *Lie wedges of endomorphism semigroups of standard subspaces in real simple Lie algebras*
15:30 – 15:55	Coffee/Tea
15:55 – 16:20	Tobias Diez *Normal Form of Equivariant Maps in Infinite Dimensions*
16:25 – 16:50	Sam Claerebout *Fock model and Segal-Bargmann transform for the orthosymplectic Lie superalgebra*
17:00	Bus shuttle to Heinz-Nixdorf-Museumsforum
17:30 – 19:30	Excursion to Heinz-Nixdorf-Museumsforum
19:30 – 23:00	Conference Dinner at Schützenhof

Saturday, February 15

9:00 – 9:45	Manfred Einsiedler *Measure Rigidity for Diagonal Actions and Applications*
9:50 – 10:15	Benjamin Küster *The frame flow on hyperbolic 3-manifolds*
10:20 – 10:45	Nguyen-Thi Dang *Topological dynamics of the Weyl chamber flow*
10:45 – 11:15	Coffee/Tea
11:15 – 12:00	Ralf Köhl *Kac–Moody symmetric spaces*
12:05 – 12:30	Robin Lautenbacher *Extending generalized spin representations*
12:30 – 13:30	Lunch
13:30 – 14:15	Tobias Hartnick *Discrete subsets of Lie groups*
14:20 – 15:05	Alessandra Iozzi *The real spectrum compactification of character varieties: Characterizations and applications*

Abstracts

Friday, February 14

Tempered homogeneous spaces

Yves Benoist
(Joint work with T. Kobayashi)
CNRS and Paris-Saclay University

Let G be a semisimple complex Lie group, H a connected complex Lie subgroup and $\mathfrak{g}$, $\mathfrak{h}$ their Lie algebras. With T. Kobayashi, we claim that the regular representation of G in $L^2(G/H)$ is tempered if and only if the orthogonal of $\mathfrak{h}$ in $\mathfrak{g}$ contains regular elements.

Discrete series representations for real spherical spaces

Job Kuit
(Joint work with Friedrich Knop, Bernhard Krötz, Eric Opdam and Henrik Schlichtkrull)
Paderborn University

Let $Z = G/H$ be a homogeneous space attached to a real reductive group G and a closed subgroup H. A principal objective in the harmonic analysis of Z is the understanding of the G-equivariant spectral decomposition of the space $L^2(Z)$ of square integrable half-densities. The irreducible components of $L^2(Z)$ are of particular interest, they comprise the discrete series for Z. In this talk I will focus on real spherical homogeneous spaces Z and present some recent results.

Fourier inversion of Whittaker functions

Erik van den Ban
Utrecht University

We will present a new Fourier inversion formula for Whittaker functions on a real reductive Lie group. We will explain how this is related to the work of Harish-Chandra [1] and of Wallach [2]. The needed theory of wave packets will be explained.

References

[1] Harish-Chandra, *The theory of the Whittaker integral*, Chapter III in Collected papers V (Posthumous), eds. R. Gangolli and V. S. Varadarajan, with assistance from J.A.C. Kolk, Springer, 2018.

[2] N.R. Wallach, *The Whittaker Plancherel Theorem*, Chapter 15 in Real reductive groups II. Acad. Press, 1992.

Holonomy properties of Riemannian homogeneous spaces

Ilka Agricola
Philipps-Universität Marburg

Famous classical holonomy theorems for Riemannian manifolds include Berger's classification of possible Riemannian holonomy groups and the holonomy characterisation of homogeneous structures by Ambrose and Singer.

In this gentle survey, I will expose recent holonomy theorems of a different kind and explain how they yield a new look on different classes of homogeneous non-symmetric Riemannian manifolds, and how the difficulties of classifying them can thus be overcome.

ODE-closed function spaces and groups of diffeomorphisms on $\mathbb{R}^d$

Armin Rainer
(Joint work with David Nenning)
Universität Wien

In this talk I will consider the flow of time-dependent vector fields on $\mathbb{R}^d$ in certain function spaces which are continuously embedded in C^1_b (the space of globally bounded C^1-functions with globally bounded derivative). The corresponding Trouvé group (which was introduced in image analysis and computational anatomy) consists of all diffeomorphisms of $\mathbb{R}^d$ which arise as flows at time 1 of vector fields of the given class. We are interested in the structure of the Trouvé group and in the regularity of its elements.

In the talk I will focus on the case of time-dependent global Hölder ($C^{n,\alpha}_b$) vector fields. This case is particulary interesting, because composition is not continuous on $C_b^{n,\alpha}$. Nevertheless, we prove that the elements of the corresponding Trouvé group are of class $\mathrm{Id}_{\mathbb{R}^d} + C_b^{n,\alpha}$; we say that the function space $C_b^{n,\alpha}$ is ODE-closed. Furthermore, we show that the Trouvé group coincides with the connected component of the identity of the group of orientation preserving diffeomorphisms of $\mathbb{R}^d$ which differ from the identity by a $C_b^{n,\alpha}$-mapping. The latter is not a topological group, since left translations are in general discontinuous. I will also discuss regularity of the flow mapping (which sends vector fields to flows).

It is interesting to compare the Hölder case with the Sobolev case H^s (for $s > d/2 + 1$), where the corresponding diffeomorphism group is a topological group and which is based on very different methods. If time permits I will also discuss further related questions.

Unitary branching laws for real reductive groups

Clemens Weiske
Aarhus University

Let G be a real reductive group, P a minimal parabolic and H a reductive subgroup of G. Unitary branching laws describe how an irreducible unitary representation of G decomposes into a direct integral of irreducible unitary representations of H when restricted to the subgroup H. If the representation is a unitary principal series representation and H has an open orbit on the flag manifold G/P, Mackey theory reduces this problem to the Plancherel formula of a homogeneous space for H which is known in many cases. We will show how to construct branching laws for other unitary representations like complementary series representations from the ones for the unitary principal series by an analytic continuation procedure and show examples for real reductive groups of rank one.

Lie wedges of endomorphism semigroups of standard subspaces in real simple Lie algebras

Daniel Oeh
Friedrich–Alexander University Erlangen–Nürnberg

Let $(\mathfrak{g}, \tau)$ be a real simple symmetric Lie algebra and let $W \subset \mathfrak{g}$ be an $e^{\mathrm{ad}\,\mathfrak{g}}$-invariant closed convex cone which is pointed and generating with $\tau(W) = -W$. For elements $h \in \mathfrak{g}$ with $\tau(h) = h$, we classify the Lie algebras $\mathfrak{g}(\tau, h)$

which are generated by the closed convex cones

$$C_{\pm}(h) := (\pm W) \cap \mathfrak{g}_{\pm 1}^{-\tau}(h),$$

where $\mathfrak{g}_{\pm 1}^{-\tau}(h) := \{x \in \mathfrak{g} : \tau(x) = -x, [h, x] = \pm x\}$. These cones occur naturally as the skew-symmetric parts of the Lie wedges of endomorphism semigroups of certain standard subspaces. We prove in particular that $\mathfrak{g}(\tau, h)$ is either a hermitian simple Lie algebra of tube type or a direct sum of two Lie algebras of this type. Moreover, we give for each hermitian simple Lie algebra and each equivalence class of involutive automorphisms τ of $\mathfrak{g}$ with $\tau(W) = -W$ a list of possible subalgebras $\mathfrak{g}(\tau, h)$ up to isomorphy.

Normal Form of Equivariant Maps in Infinite Dimensions

Tobias Diez
(Joint work with Tudor Ratiu)
TU Delft, Netherlands

Inspired by questions about the geometry of moduli spaces and symplectic quotients, I will present a general approach to normal forms of smooth equivariant maps. In the first part of the talk, I will show that a smooth map between infinite-dimensional manifolds can be brought into a certain normal form. This result generalizes and unifies the level set theorem, the constant rank theorem and the immersion theorem. In the second part of the talk, I will show how this normal form can be combined with the slice theorem to yield a normal form for an equivariant map. As an application, the equivariant normal form will be used to show that subquotients are endowed with a Kuranishi structure in a very general setting. The main results are obtained for Banach manifolds and in the tame Fréchet category using the Nash-Moser inverse function theorem.

Fock model and Segal-Bargmann transform for the orthosymplectic Lie superalgebra

Sam Claerebout
(Joint work with Sigiswald Barbier and Hendrik De Bie)
UGent

The classical Segal-Bargmann transform is an integral transform between the Schrödinger space of square-integrable functions and the Fock space of holomorphic functions.
The Segal-Bargmann transform was reinterpreted by Hilgert-Kobayashi-Möllers-Ørsted as an intertwining operator between realisations on the Schrödinger and the Fock space of the minimal representation of a Lie group.
In this talk I will give a generalisation of this approach to superspaces in order to obtain a Segal-Bargmann transform as an integral transform that intertwines the Schrödinger and Fock model for the orthosymplectic Lie superalgebra $\mathfrak{osp}(m, 2|2n)$.

Saturday, February 15

Measure Rigidity for Diagonal Actions and Applications

Manfred Einsiedler
(Joint work with Anatole Katok, Elon Lindenstrauss, Philippe Michel, and Akshay Venkatesh)
ETH Zürich

As was conjectured by Furstenberg and Margulis higher rank diagonalizable actions on homogeneous spaces should satisfy remarkable rigidity properties. We will explain some old and some more recent measure rigdity theorems in this direction as well as some of its applications to number theoretic problems.

The frame flow on hyperbolic 3-manifolds

Benjamin Küster
(Joint work with Colin Guillarmou)
Université Paris-Saclay

If $F\mathcal{M}$ denotes the bundle of oriented orthonormal frames of the tangent bundle $T\mathcal{M}$ of an oriented n-dimensional Riemannian manifold $\mathcal{M}$, the *frame flow* φ_t on $F\mathcal{M}$ is defined as follows: For $x \in \mathcal{M}$ and an n-tuple $(e_1, \dots, e_n) \in T_x\mathcal{M}$ defining an oriented orthonormal frame, $\varphi_t(e_1, \dots, e_n)$ is the n-tuple whose j-th entry is the parallel transport of e_j in the direction e_1 along the geodesic flow for time t. The map $F\mathcal{M} \ni (e_1, \dots, e_n) \mapsto e_1 \in S\mathcal{M}$ gives $F\mathcal{M}$ the structure of a fiber bundle over the unit sphere bundle $S\mathcal{M} \subset T\mathcal{M}$, and the frame flow is a lift of the geodesic flow on $S\mathcal{M}$ along this map. When $\mathcal{M}$ is compact and has strictly negative sectional curvatures, the geodesic flow is hyperbolic (Anosov) and the frame flow is partially hyperbolic: the fiber directions in the fiber bundle are additional neutral directions besides the flow direction. We will consider the situation that $\mathcal{M} = \Gamma\backslash\mathbb{H}^3 = \Gamma\backslash G/K$ is a compact oriented hyperbolic 3-manifold, where $G = \mathrm{PSO}(1,3) = KAN$ is an Iwasawa decomposition and $\Gamma \subset G$ is a cocompact torsion-free discrete subgroup. Then $F\mathcal{M} = \Gamma\backslash G$ and $S\mathcal{M} = \Gamma\backslash G/M$, where $M = Z_K(A) \cong \mathrm{SO}(2)$, and the frame and geodesic flows are given by the right-multiplication actions of the abelian group A on $\Gamma\backslash G$ and $\Gamma\backslash G/M$, respectively. In particular, the fiber bundle $F\mathcal{M}$ is a principal $\mathrm{SO}(2)$-bundle over $S\mathcal{M}$.

I will present a recent result on the existence of a spectral gap in the resonance spectrum of the generator of the frame flow, based on Fourier analysis and a semiclassical calculus for line bundle tensor powers.

Topological dynamics of the Weyl chamber flow

Nguyen-Thi Dang
Universität Heidelberg

Let G be a connected, real linear, semisimple Lie group without compact factors (not necessarily of real rank one) and Γ be a Zariski dense subgroup of G, not necessarily a lattice. I am interested in the action $\Gamma\backslash G \curvearrowleft \phi^t$, where ϕ^t is a subgroup of a Cartan subgroup of G.

Kac–Moody symmetric spaces

Ralf Köhl
JLU Gießen

Kac–Moody groups have been studied intensively in the 1980s as groups of automorphisms of Kac–Moody algebras, as generalizations of split algebraic/Chevalley groups, and as groups with a BN-pair/RGD system and, thus, with a strongly transitive action on a twin building.

Recently, Freyn, Hartnick, Horn and the author defined split real Kac–Moody symmetric spaces. Similar to the finite-dimensional situation they can be described in the coset model G/K endowed with the quotient topology induced by the Kac–Peterson topology on a split real Kac–Moody group G, in the model of all conjugates of the Cartan–Chevalley involution θ of G endowed with the subspace topology induced by the natural topoloy on the semi-direct product $G.\langle\theta\rangle$, and in the model of symmetric elements $\{g\theta(g)^{-1} \mid g \in G\}$ endowed with the subspace topology induced by G – these three models are topologically isomorphic as reflection spaces.

Unlike the finite-dimensional situation, Kac–Moody symmetric spaces have preferred directions to infinity, and in these directions one finds the twin building.

Grüning and the author proved that, if the Dynkin diagram of the Kac–Moody group is two-spherical, then the Kac–Moody symmetric space is a colimit in the category of reflection spaces of the rank-1 and rank-2 symmeric subspaces embedded at one fixed base point, a result that also applies to Riemannian symmetric spaces of split non-compact type.

It is currently unknown whether the action of G on its Kac–Moody symmetric is primitive (which would imply G modulo its center to be simple as an abstract group); for this one needs to establish that K is a maximal subgroup of G. It is also unknown whether the causal structure provided by moving towards the positive half of the twin buiding at infinity actually defines a partial order; for this one needs to establish an analogue of Kostant's convexity result for Kac–Moody groups.

Extending generalized spin representations

Robin Lautenbacher
(Joint work with Ralf Köhl)
Justus Liebig Universität Gießen

Split-real Kac-Moody algebras $\mathfrak{g}(A)(\mathbb{R})$ of symmetrizable type A posses a so-called "maximal compact subalgebra" $\mathfrak{k}(A)$ which is the fixed-point set of the Cartan-Chevalley involution of $\mathfrak{g}$. This subalgebra $\mathfrak{k}$ is in general not of Kac-Moody type unless A is spherical. I will review a certain finite-dimensional representation of $\mathfrak{k}$ - *the generalized spin representation* - for the special case of simply-laced A and present a way to extend this representation via the Weyl group W of $\mathfrak{g}$.

Discrete subsets of Lie groups

Tobias Hartnick
(Joint work with Michael Björklund)
KIT

Discrete subgroups of Lie groups play a major role in representation theory, harmonic analysis, measurable and geometric group theory and other related areas. It turns out that large parts of the theory still work for more general *subsets* of Lie groups which satisfy sufficiently strong discreteness conditions. Such subsets automatically have a group-like structure, and these group-like structures have been studied in various contexts (additive combinatorics, aperiodic order, superstrong approximation , Property (T)) for some time. The key difference between the study of discrete subgroups and more general discrete subsets is that the homogeneous space G/Γ associated with a discrete subgroup Γ of a Lie group G is replaced by a non-homogeneous dynamical system. We explain how these dynamical systems can be used to extend various classical results concerning discrete subgroups to more general discrete subsets.

The real spectrum compactification of character varieties: Characterizations and applications

Alessandra Iozzi
(Joint work with Marc Burger, Anne Parreau, and Maria Beatrice Pozzetti)
ETHZ

We describe properties of a compactification of general character varieties with good topological properties and give various interpretations of its ideal points. We relate this to the Thurston-Parreau compactification and apply our results to the theory of maximal representations.

List of Participants

Agricola	Ilka	Philipps-Universitaet Marburg
Arends	Christian	Universität Paderborn
Budde	Julia	Universität Paderborn
Bux	Kai-Uwe	Universität Bielefeld
Claerebout	Sam	UGent
Dahmen	Rafael	Karlsruher Institut für Technologie (KIT)
Dang	Nguyen-Thi	Universität Heidelberg
Diez	Tobias	TU Delft
Einsiedler	Manfred	ETH Zurich
Glöckner	Helge	Universität Paderborn
Grüning	Julius	Justus-Liebig-Universität Gießen
Hanusch	Maximilian	Universität Paderborn
Hartnick	Tobias	KIT
Hilgert	Joachim	Universität Paderborn
Iozzi	Alessandra	ETHZ
Januszewski	Fabian	Universität Paderborn
Köhl	Ralf	JLU Gießen
Kuit	Job	Universität Paderborn
Küster	Benjamin	Université Paris-Sud
Lautenbacher	Robin	Justus-Liebig-Universität Gießen
Neeb	Karl-Hermann	FAU Erlangen-Nuernberg
Oeh	Daniel Friedrich-Alexander	Universität Erlangen-Nürnberg
Rainer	Armin	Universität Wien
Ramacher	Pablo	Marburg University
Rösler	Margit	Institut für Mathematik, Uni Paderborn
Schober	Jonas	Friedrich-Alexander-Universität Erlangen-Nürnberg
Schoemann	Claudia	Universität Hannover
Skill	Thomas	University of Applied Sciences Bochum
van den Ban	Erik	Utrecht University
Vock	Christian	JLU Gießen
Weich	Tobias	Universität Paderborn
Weiske	Clemens	Aarhus University
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Yirtici	Cigdem	University of Stuttgart