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FAKULTÄT
FÜR MATHEMATIK, INFORMATIK
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FACHBEREICH MATHEMATIK

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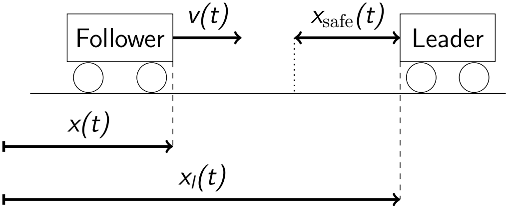
A Universal Model-Free and Safe Adaptive Cruise Control Mechanism

Vehicle following framework



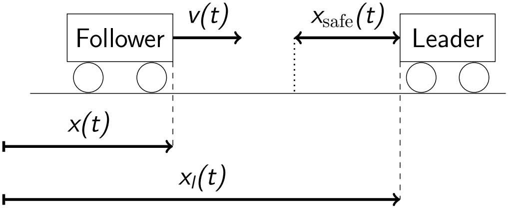
$$\dot{x}(t) = v(t), \quad m\dot{v}(t) = u(t) - F_g(t) - F_a\big(t, v(t)\big) - F_r\big(v(t)\big)$$

forces due to gravity: $F_g(t) = mg\sin\theta(t)$

aerodynamic drag : $F_a(t, v) = \frac{1}{2}\rho(t)C_d A v^2$

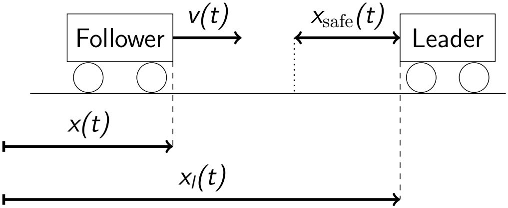
rolling friction : $F_r(v) = mgC_r \operatorname{sgn}(v)$

Control objective



given: favourite speed $v_{\mathrm{ref}}(t)$, safety distance $x_{\mathrm{safe}}(t) = \lambda_1 v(t) + \lambda_2$

Control objective



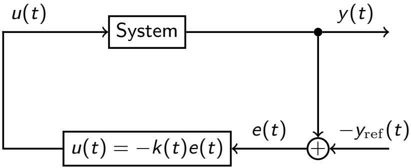
given: favourite speed $v_{\text{ref}}(t)$, safety distance $x_{\text{safe}}(t) = \lambda_1 v(t) + \lambda_2$

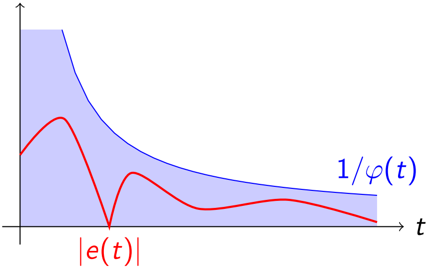
$$\textbf{Aim:} \quad u(t) = F\big(t, v(t), x_l(t) - x(t)\big) \quad \text{s.t.}$$

(O1) $x_l(t) - x(t) \geq x_{\text{safe}}(t)$,

(O2) $|v(t) - v_{\text{ref}}(t)|$ is as small as possible such that (O1) is not violated.

Funnel Control



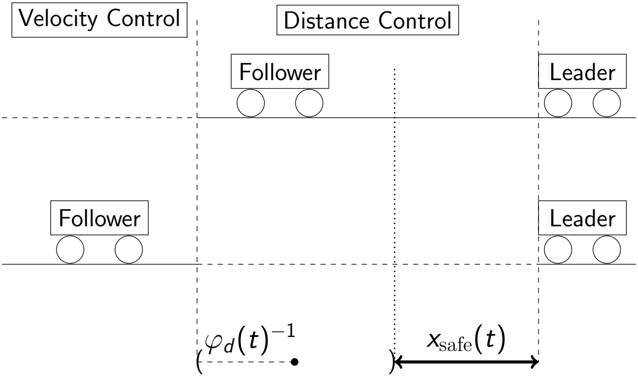


$$k(t) = \frac{1}{1 - \varphi(t)^2 e(t)^2}$$

[Ilchmann, Ryan, Sangwin '02]:
Works, if

- relative degree $= 1$
- stable internal dynamics

Funnel cruise control



Velocity funnel control

tracking error: $e_v(t) = v(t) - v_{\mathrm{ref}}(t) \in \mathcal{F}_v(t) := \left(-\varphi_v(t)^{-1}, \varphi_v(t)^{-1}\right)$

$\rightarrow$ treat $v(t)$ as output of the system

$$\dot{v}(t) = \tfrac{1}{m} u(t) + f_v\big(t, v(t)\big)$$

controller:

$$u_v(t) = -k_v(t) e_v(t),$$
$$k_v(t) = \frac{1}{1 - \varphi_v(t)^2 e_v(t)^2}$$

Distance funnel control

tracking error:
$$e_d(t) = x(t) - x_l(t) + x_{\mathrm{safe}}(t) + \varphi_d(t)^{-1} \in \mathcal{F}_d(t) := \left(-\varphi_d(t)^{-1}, \varphi_d(t)^{-1}\right)$$

Distance funnel control

tracking error: $$e_d(t) = x(t) - x_l(t) + x_{\mathrm{safe}}(t) + \varphi_d(t)^{-1} \in \mathcal{F}_d(t) := \left(-\varphi_d(t)^{-1}, \varphi_d(t)^{-1}\right)$$

output: $$y(t) = \lambda_1 v(t) + x(t),$$

reference signal: $$y_{\mathrm{ref}}(t) = x_l(t) - \lambda_2 - \varphi_d(t)^{-1}$$

Distance funnel control

tracking error: $$e_d(t) = x(t) - x_l(t) + x_{\mathrm{safe}}(t) + \varphi_d(t)^{-1} \in \mathcal{F}_d(t) := \left(-\varphi_d(t)^{-1}, \varphi_d(t)^{-1}\right)$$

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$$\dot{x}(t) = v(t) = -\tfrac{1}{\lambda_1} x(t) + \tfrac{1}{\lambda_1} y(t),$$
$$\dot{y}(t) = \tfrac{\lambda_1}{m} u(t) + f_d\big(t, x(t), y(t)\big)$$

controller: $$u_d(t) = -k_d(t) e_d(t),$$
$$k_d(t) = \frac{1}{1 - \varphi_d(t)^2 e_d(t)^2}$$

Final control design

Intuitive approach:

- if $e_d(t) = -\varphi_d(t)^{-1}$, then switch from u_v to u_d and vice versa
- while u_d is active it should be possible that $e_v(t) < -\varphi_v(t)^{-1}$, but not $e_v(t) \geq \varphi_v(t)^{-1}$

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Problem:

- when $e_d(t) \to -\varphi_d(t)^{-1}$ we have $k_d(t) \nearrow \infty$
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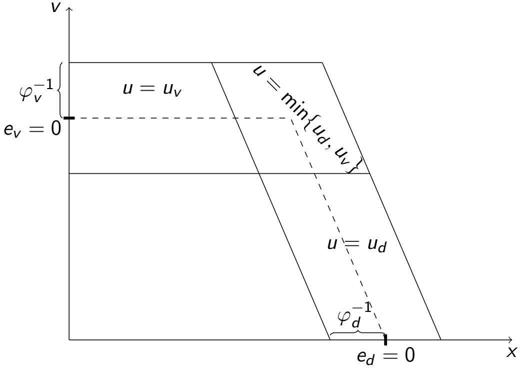

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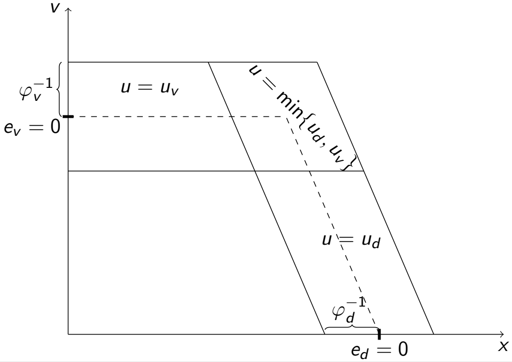
- when $e_v(t) \to -\varphi_v(t)^{-1}$ we have $k_v(t) \nearrow \infty$

Final control design

$$u(t) = \begin{cases} u_v(t), & e_d(t) \le -\varphi_d(t)^{-1} \ \wedge \ e_v(t) \in \mathcal{F}_v(t), \\ u_d(t), & e_v(t) \le -\varphi_v(t)^{-1} \ \wedge \ e_d(t) \in \mathcal{F}_d(t), \\ \min\{u_v(t), u_d(t)\}, & e_v(t) \in \mathcal{F}_v(t) \ \wedge \ e_d(t) \in \mathcal{F}_d(t), \end{cases}$$



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Theorem [B., Rauert '18]

$v_{\text{ref}}, x_l \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R})$, $x_{\text{safe}} = \lambda_1 v(t) + \lambda_2$
$\implies x, v, u \in L^\infty$ and

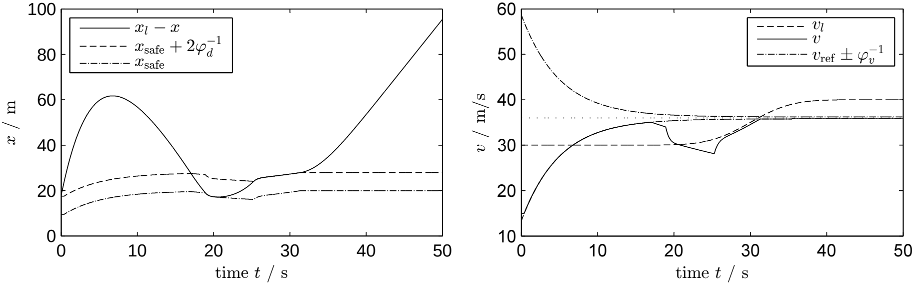
$$\varphi_v(t)^{-1} - e_v(t) \geq \varepsilon, \quad \varphi_d(t)^{-1} - e_d(t) \geq \varepsilon,$$
$$\max\{0, \varphi_v(t)^{-1} + e_v(t)\} + \max\{0, \varphi_d(t)^{-1} + e_d(t)\} \geq \varepsilon$$

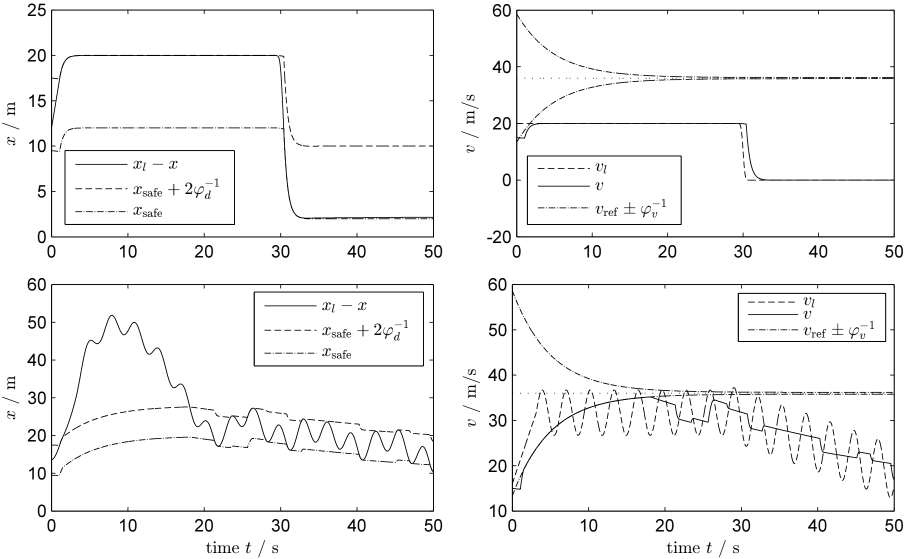
Simulation

m	$\theta(t)$	$\rho(t)$	C_d	C_r	A
1300 kg	0 rad	1.3 kg/m^3	0.32	0.01	2.4 m^2

x^0	v^0	λ_1	λ_2	v_{ref}
0 m	15 m s^{-1}	0.5 s	2 m	36 m s^{-1}

$$\varphi_v(t) = \left(22.5e^{-0.2t} + 0.2\right)^{-1}, \quad \varphi_d(t) = 0.25.$$




x / m
$x_l - x$
$x_{\text{safe}} + 2\varphi_d^{-1}$
x_{safe}
v / m/s
v_l
v
$v_{\text{ref}} \pm \varphi_v^{-1}$
time t / s