Funnel control for electrical circuits

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System class

We consider passive linear time-invariant RLC circuits which are modeled by modified nodal analysis as a differential-algebraic system of the form

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t),
\]

(1)

where

\[
sE - A = \begin{bmatrix}
sA_L C A_L^T + A_K GA_K & A_K \cr -A_K^T & sL & 0 \\
-A_L^T & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad B = C^T = \begin{bmatrix}
-A_Z & 0 \\
0 & 0 \\
0 & -I_n
\end{bmatrix},
\]

\[x = (\eta^T, i^T, v^T)^T, \quad u = (i^T, v^T)^T, \quad y = (-v^T, -i^T)^T, \]

and

\[C \in \mathbb{R}^{n \times n}, \quad G \in \mathbb{R}^{n \times m}, \quad L \in \mathbb{R}^{n \times n}, \quad A_L \in \mathbb{R}^{n \times n}, \quad A_K \in \mathbb{R}^{n \times m}, \quad A_V \in \mathbb{R}^{n \times n}, \quad A_T \in \mathbb{R}^{n \times n}, \]

\[n = n_L + n_V, \quad m = n_L + n_V, \quad \eta(t) \text{ vector of node potentials}, \]

\[i_L(t), i_V(t), i_T(t) \text{ vectors of currents through inductances, voltage and current sources}, \]

\[v_L(t), v_V(t) \text{ voltages of voltage and current sources}.\]

Passivity of the system implies

\[C = C^T > 0, \quad L = L^T > 0, \quad G + G^T > 0.\]

Control objective

\[
\phi \in C^\infty([0, \infty); \mathbb{R}) \text{ s.t. } \phi, \phi \phi \text{ are bounded, } \phi(0) = 0, \phi(t) > 0 \text{ for } t > 0 \text{ and } \liminf_{t \to \infty} \phi(t) > 0 \text{ (the set of these functions is denoted by } \Phi); \]

\[
y_{ref} \in B^\infty([0, \infty); S), \quad \text{i.e., } y_{ref} \in C^\infty([0, \infty); \mathbb{R}) \text{ and } y_{ref}^{(k)} \text{ is bounded for all } k \geq 0, \quad S \subseteq \mathbb{R}^m \text{ is a subspace}.\]

We seek that the tracking error \( e = y - y_{ref} \) evolves within the performance funnel \( F_\phi := \{ (t, e) \in \mathbb{R} \times \mathbb{R}^m | \phi(t) \| e \| \leq 1 \}. \) To ensure error evolution within the funnel, we introduce the funnel controller:

\[
u(t) = -k(t) e(t), \quad k(t) = \frac{1}{1 - \phi(t)^2 \| e(t) \|^2},
\]

(2)

We call \( \lambda \in \mathbb{C} \) an invariant zero of (1) if

\[
\text{rk}_C [AE - A\lambda B - C 0] < \text{rk}_R [sE - A\lambda B - C 0].
\]

Main result

Theorem. Assume that all invariant zeros of (1) are in \( \mathbb{C}_- \). Let \( Z_{RCL} \) be a matrix with full column rank such that

\[\text{im } Z_{RCL} = \ker [A_L A_K A_T A_V]^T\]

and let \( y_{ref} \) be a reference trajectory satisfying

\[y_{ref} \in B^\infty ([0, \infty); \mathbb{R}^m \times \ker Z_{RCL}^TA_V).\]

Let \( \phi \in \Phi \) and \( x^0 \in \mathbb{R}^m \) be a consistent initial value. Then (2) applied to (1) has a global solution \( x \in L^\infty, k \in L^\infty \) such that

\[\exists \varepsilon > 0 \forall t > 0 : \| e(t) \| \leq \phi(t)^{-1} \varepsilon.\]

Simulation

\[n = 50, \quad \phi = \sin, \cos, \quad \text{and } y_{ref} = (\sin, \cos)^T, \quad t \mapsto 0.5 t e^{-t} + 2 \arctan t\]

Reference: T. Berger and T. Reis, Zero dynamics and funnel control for linear electrical circuits, Hamburger Beiträge zur Angewandten Mathematik 2013-03

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