Funnel control of general linear DAE systems

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Abstract

We study linear differential-algebraic multi-input, multi-output systems which are not necessarily regular and present necessary conditions for feasibility of funnel control. Asymptotic stability of the zero dynamics is the fundamental assumption to guarantee that the funnel controller (that is a static nonlinear output error feedback) achieves tracking of a reference signal by the output signal within a pre-specified performance funnel.

System class

We consider the class of systems governed by the equation

$$\begin{aligned} E\,\dot x(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t)\,, \end{aligned} \tag{1}$$

where $E, A \in \mathbb{R}^{l\times n}$, $B \in \mathbb{R}^{l\times m}$, $C \in \mathbb{R}^{p\times n}$. The set of these systems is denoted by $\Sigma_{l,n,m,p}$ and we write $[E,A,B,C] \in \Sigma_{l,n,m,p}$.

A trajectory $(x,u,y) : \mathbb{R} \to \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p$ is said to be a *solution* of (1) if, and only if, it belongs to the *behaviour* of (1):

$$\mathcal{B}_{(1)} := \left\{ (x,u,y) \in \mathcal{C}^1(\mathbb{R};\mathbb{R}^n) \times \mathcal{C}(\mathbb{R};\mathbb{R}^m) \times \mathcal{C}(\mathbb{R};\mathbb{R}^p) \;\middle|\; (x,u,y) \text{ solves (1) for all } t \in \mathbb{R} \right\}$$

The zero dynamics

The *zero dynamics* of system (1) are defined as the set of trajectories

$$\mathcal{ZD}_{(1)} := \left\{ (x,u,y) \in \mathcal{B}_{(1)} \;\middle|\; y = 0 \right\}.$$

The zero dynamics are *asymptotically stable* if, and only if,

$$\forall\,(x,u,y) \in \mathcal{ZD}_{(1)} : \ \lim_{t\to\infty} (x(t), u(t)) = (0,0).$$

Control objective

Given, for $\mu \in \mathbb{N}$,

- $\varphi \in \mathcal{C}^\mu(\mathbb{R}_{\ge 0};\mathbb{R})$ s.t. $\varphi, \dot\varphi$ are bounded, $\varphi(0) = 0$, $\varphi(t) > 0$ for $t > 0$ and $\liminf_{t\to\infty} \varphi(t) > 0$ (the set of these functions is denoted by Φ^μ);
- a reference signal $y_{\rm ref} \in \mathcal{C}^{\mu+1}(\mathbb{R}_{\ge 0};\mathbb{R}^m)$ s.t. $y_{\rm ref}^{(k)}$ is bounded for $k = 0,\ldots,\mu+1$ (the set of these functions is denoted by $\mathcal{Y}^\mu$).

We seek that the tracking error $e = y - y_{\rm ref}$ evolves within the performance funnel $\mathcal{F}_\varphi := \left\{ (t,e) \in \mathbb{R}_{\ge 0} \times \mathbb{R}^m \;\middle|\; \varphi(t)\|e\| < 1 \right\}$.

To ensure error evolution within the funnel, we introduce, for $\hat k > 0$, the *funnel controller*:

$$\boxed{\begin{aligned} u(t) &= -k(t)\,e(t), \qquad \text{where} \qquad e(t) = y(t) - y_{\rm ref}(t) \\ k(t) &= \frac{\hat k}{1 - \varphi(t)^2\|e(t)\|^2}. \end{aligned}} \tag{2}$$

Main result

Theorem. Let $[E,A,B,C] \in \Sigma_{n,n,m,m}$ with asymptotically stable zero dynamics and $\operatorname{rk} C = m$. Suppose that, for the inverse $L(s)$ of $\begin{bmatrix} sE-A & -B \\ -C & 0 \end{bmatrix}$ over $\mathbb{R}(s)$, the matrix

$$\Gamma = -\lim_{s\to\infty} s^{-1}[0, I_m]L(s)\begin{bmatrix} 0 \\ I_m \end{bmatrix} \in \mathbb{R}^{m\times m}$$

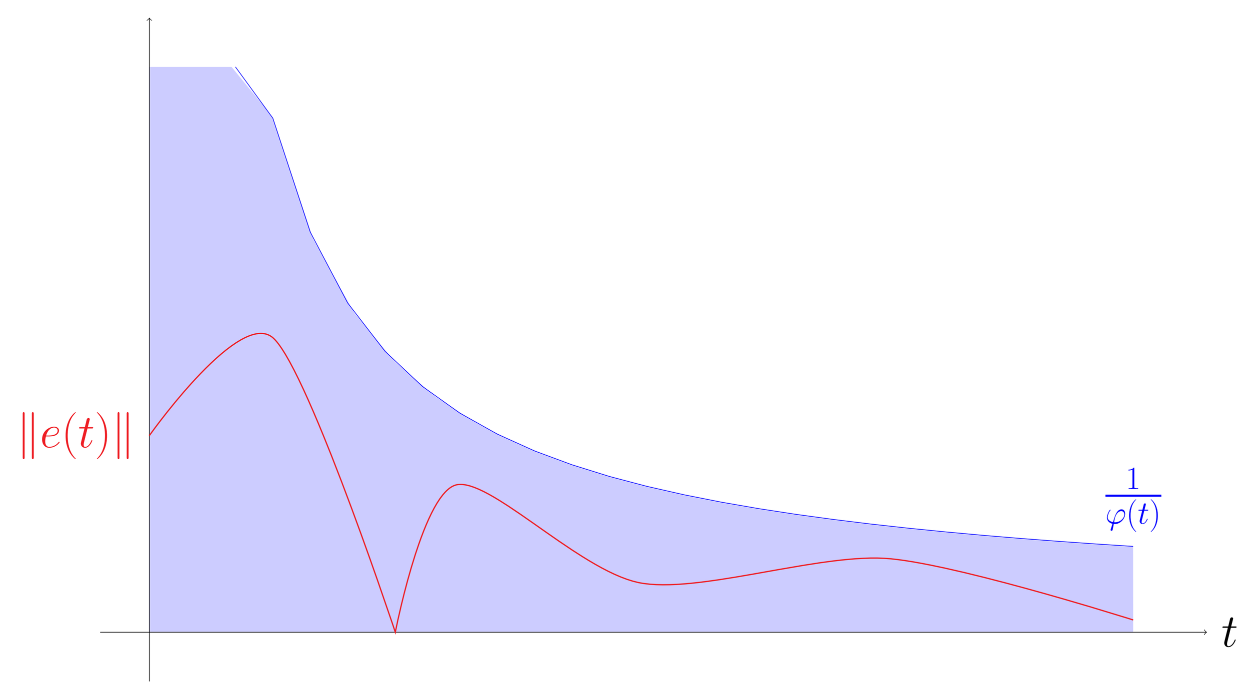
exists and satisfies $\Gamma = \Gamma^\top \ge 0$. Let, for $\mu \in \mathbb{N}$ sufficiently large, $\varphi \in \Phi^\mu$ define a performance funnel $\mathcal{F}_\varphi$. Then, for any reference signal $y_{\rm ref} \in \mathcal{Y}^\mu$, any consistent initial value $x^0 \in \mathbb{R}^n$, and initial gain

$$\hat k > \left\| \lim_{s\to\infty} \left([0, I_m]L(s)\begin{bmatrix} 0 \\ I_m \end{bmatrix} + s\Gamma \right) \right\|,$$

the application of the funnel controller (2) to (1) yields a closed-loop initial-value problem that has a solution and every solution can be extended to a global solution. Furthermore, for every global solution x,

(i) x is bounded and the corresponding tracking error $e = Cx - y_{\rm ref}$ evolves uniformly within the performance funnel $\mathcal{F}_\varphi$; more precisely: there exists $\varepsilon > 0$ such that for all $t > 0$ we have $\|e(t)\| \le \varphi(t)^{-1} - \varepsilon$.

(ii) the corresponding gain function k given by (2) is bounded.



References

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Personal Information

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